Notes on Number Theory and Discrete Mathematics
Print ISSN 1310–5132, Online ISSN 2367–8275
2026, Volume 32, Number 3, 629–631
DOI: 10.7546/nntdm.2026.32.3.629-631

A short remark on a connection between Goldbach's conjecture and quadratic forms

Mladen V. Vassilev-Missana

5 V. Hugo Str., Sofia-1504, Bulgaria
e-mail: missana@abv.bg

Received: 20 October 2026 **Revised:** 26 October 2026
Accepted: 29 October 2026 **Online First:** 31 October 2026

Abstract: In the paper, a new version of the binary/strong Goldbach's conjecture is considered and discussed. It is based on quadratic forms representation and on a new conjecture, that establishes equivalence.
Keywords: Binary/strong Goldbach's conjecture, Composite number, Prime number.
2020 Mathematics Subject Classification: 11A41, 11E25, 11P32.

In memory of my wife Maria Vassileva (1956 – 2026)

1 Introduction

As it is well-known, the famous Binary/strong Goldbach's conjecture states:

G: *Every even integer greater than* 2 *is the sum of two primes* [5].

This conjecture is still unproved. In 2005, the author formulated an equivalent form of **G** in [4] and together with P. Vassilev and K. Atanassov, they formulated four other forms of **G** as hypotheses H1–H4 in [2] (see also [3]).

For the case, when n is prime, we have

$$2n = n + n$$


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and the assertion of the conjecture is obviously true. So, it is natural to consider only the case, when n is a composite number. Thus, **G** is equivalent to the following formulation.

G_1: *Let $n > 1$ be a composite number. Then there exist two different primes p and q ($p < q$) such that*

$$2n = p + q. \tag{1}$$

Of course, p and q are both odd numbers.

2 Main results

Lemma 1. *If $\mathbf{G_1}$ is true, then the numbers:*

$$n^2 - pq; \quad p^2 + 8nq; \quad q^2 + 8np$$

are perfect squares.

Proof. From (1) we obtain

$$n = \frac{p+q}{2}.$$

Hence,

$$n^2 - pq = \left(\frac{p+q}{2}\right)^2 - pq = \frac{q^2 - 2qp + p^2}{4} = \left(\frac{q-p}{2}\right)^2.$$

Also:

$$p^2 + 8nq = p^2 + 4(p+q)q = (p+2q)^2,$$
$$q^2 + 8np = q^2 + 4(p+q)p = (q+2p)^2. \qquad \square$$

We will formulate the following conjecture.

V-M: *Let p and q be odd primes and $p < q$. Then the numbers:*

$$u = p^2 + 4q(pq+1); \quad v = q^2 + 4p(pq+1)$$

cannot be perfect squares simultaneously.

Now, we are ready to prove the following assertion that is a new version of $\mathbf{G_1}$, using quadratic forms. Let us assume that the conjecture **V-M** is true. Then:

G_2: *For any composite $n > 1$ there exist odd primes p and q ($p < q$), such that the numbers:*

$$n^2 - pq: \quad p^2 + 8nq; \quad q^2 + 8np$$

are all perfect squares. Then (1) *holds.*

Indeed, let

$$n^2 - pq = a^2; \quad p^2 + 8nq = b^2; \quad q^2 + 8np = c^2$$

for some positive integers a, b, c.

From the first equality we obtain:

$$pq = n^2 - a^2 = (n - a)(n + a). \tag{2}$$

Since $pq = n^2 - a^2 > 0$, we have $0 < a < n$, and hence $n - a$ must be a positive integer.

If $n - a = 1$, then

$$n = \frac{pq + 1}{2}.$$

Therefore,

$$b^2 = p^2 + 8nq = p^2 + 4q(pq + 1),$$
$$c^2 = q^2 + 8np = q^2 + 4p(pq + 1).$$

But the last equalities contradict **V-M**. So, the equality $n - a = 1$ is impossible. Therefore,

$$n - a > 1. \tag{3}$$

Since p and q are odd primes and $p < q$, then (2) yields

$$p = n - a; \quad q = n + a.$$

Hence,

$$2n = p + q$$

and (1) is proved.

Therefore, we obtain the main result of the paper.

Theorem 1. *If the* **V-M** *conjecture is true, then* $\mathbf{G_1}$ *and* $\mathbf{G_2}$ *are equivalent.*

Remark. In [1], 18 years after [2], K. Sasikumar reformulated hypothesis H1 from there without an appropriate citation, yet in an incorrect form and with an error in the proof.

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