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On a generalization of the Hosoya triangle

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Abstract: This paper introduces the Fibonacci polynomial triangle, inspired by the structure of the Hosoya triangle and constructed using Fibonacci polynomials. We then present and rigorously prove a series of novel identities and fundamental properties specifically associated with this Fibonacci polynomial triangle. These findings contribute to a deeper understanding of the algebraic structures and combinatorial patterns that emerge when Fibonacci polynomials are organized in such a triangular fashion, revealing new relationships and characteristics within this framework. This exploration aims to further elucidate the rich interplay between polynomial sequences and triangular constructions.
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1 Introduction

The Fibonacci numbers and their generalizations play a prominent role in mathematical sequences and structures, known for their rich properties and wide range of applications. These sequences not only offer deep insights into number theory and combinatorics, but also find applications in diverse fields such as computer science, biology, and even art.

A natural extension of this celebrated sequence is given by the *Fibonacci polynomials*. These polynomials preserve the fundamental recurrence relation of Fibonacci numbers while providing a richer algebraic framework. As such, they allow for more flexible analytical techniques and deeper algebraic investigations, extending classical results to polynomial settings.

Another important combinatorial structure associated with Fibonacci numbers is the *Hosoya triangle*, also known as the Fibonacci triangle. This triangular array resembles Pascal's triangle but is constructed using Fibonacci numbers instead of binomial coefficients. The Hosoya triangle provides both a visual and combinatorial representation of Fibonacci numbers and reveals intricate internal relationships. Various properties of this triangle have been studied extensively, including its *GCD properties* [8] and *matrix representations* [2].

The combinatorial richness of the Hosoya triangle has further been emphasized through counting arguments and identity proofs. In particular, Benjamin and Elizondo [1] presented elegant combinatorial proofs of several identities inspired by the Hosoya triangle, highlighting its role as a powerful counting structure in discrete mathematics.

Analogous triangular arrangements based on other recursive sequences have also been introduced in the literature. Notable examples include the *Lucas triangle* [15], the *Narayana triangle* [14], and the *Padovan triangle* [4]. These constructions exhibit structural and combinatorial properties similar to those of the Hosoya triangle, reinforcing the unifying role of recursive sequences in triangular arrays.

Beyond the classical Fibonacci sequence, Fibonacci-like sequences have been studied within the framework of generalized Pascal's triangles. Vincenzi and Siani [16] investigated the properties of diagonals of generalized Pascal triangles and established combinatorial relationships between Fibonacci-like sequences and the classical Fibonacci sequence. Their results provide a natural combinatorial background for the triangular constructions considered in the present work.

The study of these ideas naturally extends to the *Hosoya polynomial triangle*, in which the entries are Fibonacci polynomials rather than numerical Fibonacci values. This generalization uncovers additional algebraic and combinatorial structures. Flórez, Higuita, and Mukherjee [7] explored several remarkable patterns in this setting, including the emergence of the *Star of David* and related configurations, while alternating sum properties were investigated in [6]. Moreover, GCD-related phenomena have been extended to polynomial settings, as demonstrated in the study of the *generalized Star of David property* for generalized Hosoya triangles [5].

Further generalizations were introduced by Czabarka, Flórez, and Junes [3], who defined a discrete convolution based on the entries of the generalized Hosoya triangle. Using generating functions, they showed that sums of selected entries along rows and diagonals can be expressed as linear combinations involving Fibonacci and Lucas numbers. Their approach connects generalized Hosoya triangles with convolution structures and sequences listed in the OEIS, highlighting their broad applicability in combinatorics.

Motivated by these developments, the present study focuses on the interaction between Fibonacci polynomials and the Hosoya triangle, with particular emphasis on their polynomial generalizations. By combining algebraic, combinatorial, and geometric perspectives, we aim to extend existing results and provide new insights into the structure of the Hosoya polynomial triangle. In particular, we establish new identities and investigate structural properties arising from geometric–combinatorial configurations.

To establish a common ground for our exploration, let us define the primary sequences:

The *Fibonacci polynomial sequence* $\{f_n(x)\}$ is defined by a second-order recurrence relation:

$$f_n(x) = xf_{n-1}(x) + f_{n-2}(x), \quad n \geq 2 \tag{1}$$

with the initial conditions $f_0(x) = 0$ and $f_1(x) = 1$. The first few members of this sequence are $0, 1, x, x^2 + 1, x^3 + 2x, x^4 + 3x^2 + 1, x^5 + 4x^3 + 3x$.

A closely related sequence is the Lucas polynomials, which follow the same recurrence with different initial conditions.

The *Lucas polynomial sequence* $\{l_n(x)\}$ is defined by a second-order recurrence relation:

$$l_n(x) = xl_{n-1}(x) + l_{n-2}(x), \quad n \geq 2 \tag{2}$$

with the initial conditions $l_0(x) = 2$ and $l_1(x) = x$. The first few members of this sequence are 2, $x, x^2 + 2, x^3 + 3x, x^4 + 4x^2 + 2, x^5 + 5x^3 + 5x, x^6 + 6x^4 + 9x^2 + 2$.

The recurrence relations (1) and (2) share a common characteristic equation:

$$t^2 - xt - 1 = 0.$$

If its roots are denoted by $\lambda(x)$ and $\mu(x)$, then the following equalities hold:

$$\begin{aligned} \lambda(x) + \mu(x) &= x, \\ \lambda(x) - \mu(x) &= \sqrt{x^2 + 4}, \\ \lambda(x)\mu(x) &= -1. \end{aligned}$$

Moreover, the Binet-like formulas for the Fibonacci and Lucas polynomial sequences are:

$$f_n(x) = \frac{\lambda^n(x) - \mu^n(x)}{\lambda(x) - \mu(x)} \tag{3}$$

and

$$l_n(x) = \lambda^n(x) + \mu^n(x), \tag{4}$$

respectively. The generating function for the Fibonacci polynomial is

$$\sum_{n=0}^{\infty} f_n(x)t^n = \frac{t}{1 - xt - t^2} \tag{5}$$

2 Hosoya polynomial triangle

H. Hosoya defined a triangular array $\{f_{m,n}\}_{m\geq n\geq 0}$ of positive integers which is called Fibonacci triangle. The Fibonacci or Hosoya's triangle $\{f_{m,n}\}_{m\geq n\geq 0}$ is defined by the two recurrences

(i) $f_{m,n} = f_{m-1,n} + f_{m-2,n} \quad (m \geq 2)$
(ii) $f_{m,n} = f_{m-1,n-1} + f_{m-2,n-2} \quad (m \geq 2),$

with the initial conditions

$$f_{0,0} = 1, f_{1,0} = 1, f_{1,1} = 1, f_{2,1} = 1.$$

Hosoya demonstrated that the array $f_{m,n}$ represents a two-dimensional extension of the Fibonacci sequence (for the details, see [10–13, 17, 18]). Inspired by the structure of the Hosoya triangle, various generalizations have been studied using polynomial analogues. In particular, the Hosoya polynomial triangle introduces a polynomial version of the original Fibonacci triangle by replacing the Fibonacci numbers with Fibonacci polynomials. Notably, Flórez, Higuita, and Mukherjee presented such a generalization and investigated its combinatorial properties and identities, including alternating sums and geometric patterns such as the Star of David (see [6, 7]).

The Hosoya polynomial triangle $\left\{h_{m,n}^{(x)}\right\}_{m\geq n\geq 1}$ is defined by the two recurrence relations:

$$h_{m,n}^{(x)} = xh_{m-1,n}^{(x)} + h_{m-2,n}^{(x)} \qquad (m \geq 3), \tag{6}$$

$$h_{m,n}^{(x)} = xh_{m-1,n-1}^{(x)} + h_{m-2,n-2}^{(x)} \qquad (m \geq n \geq 3) \tag{7}$$

with the initial conditions

$$h_{1,1}^{(x)} = 1, h_{2,1}^{(x)} = x, h_{2,2}^{(x)} = x, h_{3,2}^{(x)} = x^2.$$

Here, $h_{m,n}^{(x)}$ denotes the element in row m and column n. The polynomials $h_{m,n}^{(x)}$ can be arranged triangularly as in Figure 1 or Figure 2. The arrangement in Figure 1 resembles a binomial structure but is generated by the recurrence relations of Hosoya polynomials.

$$\begin{array}{ccccccccccccc}
 & & & & & & h_{1,1}^{(x)} & & & & & & \\
 & & & & & h_{2,1}^{(x)} & & h_{2,2}^{(x)} & & & & & \\
 & & & & h_{3,1}^{(x)} & & h_{3,2}^{(x)} & & h_{3,3}^{(x)} & & & & \\
 & & & h_{4,1}^{(x)} & & h_{4,2}^{(x)} & & h_{4,3}^{(x)} & & h_{4,4}^{(x)} & & & \\
 & & h_{5,1}^{(x)} & & h_{5,2}^{(x)} & & h_{5,3}^{(x)} & & h_{5,4}^{(x)} & & h_{5,5}^{(x)} & & \\
 & h_{6,1}^{(x)} & & h_{6,2}^{(x)} & & h_{6,3}^{(x)} & & h_{6,4}^{(x)} & & h_{6,5}^{(x)} & & h_{6,6}^{(x)} & \\
h_{7,1}^{(x)} & & h_{7,2}^{(x)} & & h_{7,3}^{(x)} & & h_{7,4}^{(x)} & & h_{7,5}^{(x)} & & h_{7,6}^{(x)} & & h_{7,7}^{(x)} \\
 & \iddots & & \iddots & & \iddots & & \ddots & & \ddots & & \ddots &
\end{array}$$

Figure 1. Binomial-like arrangement of Hosoya polynomial triangle values

$$1$$

$$x \qquad x$$

$$x^2+1 \qquad x^2 \qquad x^2+1$$

$$x^3+2x \qquad x^3+x \qquad x^3+x \qquad x^3+2x$$

$$x^4+3x^2+1 \qquad x^4+2x^2 \qquad x^4+2x^2+1 \qquad x^4+2x^2 \qquad x^4+3x^2+1$$

$$x^5+4x^3+3x \qquad x^5+3x^3+x \qquad x^5+3x^3+2x \qquad x^5+3x^3+2x \qquad x^5+3x^3+x \qquad x^5+4x^3+3x$$

$$\iddots \qquad \iddots \qquad \iddots\ \ddots \qquad \ddots \qquad \ddots$$

Figure 2. Hosoya polynomial triangle values

From the recurrence relation (6) we write

$$h_{m,1}^{(x)} = xh_{m-1,1}^{(x)} + h_{m-2,1}^{(x)}$$

and by $h_{1,1}^{(x)} = 1 = f_1(x)$ and $h_{2,1}^{(x)} = x = f_2(x)$, we conclude that

$$h_{m,1}^{(x)} = f_m(x).$$

Likewise, since $h_{m,m}^{(x)} = xh_{m-1,m}^{(x)} + h_{m-2,m}^{(x)}$, it follows that

$$h_{m,m}^{(x)} = f_m(x).$$

Similarly, we can show that

$$h_{m,1}^{(x)} = h_{m,m}^{(x)} = f_m(x)$$

or

$$h_{m+1,2}^{(x)} = h_{m+1,m}^{(x)} = xf_m(x).$$

Lemma 1. *For* $0 \le k \le n-m-1$*, we have*

$$h_{m,n}^{(x)} = f_{k+1}(x)h_{m-k,n}^{(x)} + f_k(x)h_{m-k-1,n}^{(x)}. \tag{8}$$

Proof. Using the induction method, the statement is true for $k=1$ as $h_{m,n}^{(x)} = xh_{m-1,n}^{(x)} + h_{m-2,n}^{(x)}$. Assume it is true for $k=r$. Now, let us prove its validity for $k=r+1$.

$$\begin{aligned}
h_{m,n}^{(x)} &= f_{r+1}(x)h_{m-r,n}^{(x)} + f_r(x)h_{m-r-1,n}^{(x)} \\
&= f_{r+1}(x)(xh_{m-r-1,n}^{(x)} + h_{m-r-2,n}^{(x)}) + f_r(x)h_{m-r-1,n}^{(x)} \\
&= (xf_{r+1}(x) + f_r(x))h_{m-r-1,n}^{(x)} + f_{r+1}(x)h_{m-r-2,n}^{(x)} \\
&= f_{r+2}(x)h_{m-r-1,n}^{(x)} + f_{r+1}(x)h_{m-r-2,n}^{(x)}
\end{aligned}$$

This completes the inductive step and hence proves the lemma. □

The following identity is a well-known result in the literature; see [6, p. 5, Proposition 1] by Flórez, Higuita, and Mukherjee, where an equivalent formulation and a detailed proof are provided. For completeness, we record the identity here using our notation.

Proposition 2. [6] *The following equality is valid:*

$$h_{m,n}^{(x)} = f_{m-n+1}(x)f_n(x) \tag{9}$$

Consequently, every entry $h_{m,n}^{(x)}$ in the Hosoya polynomial triangle can be expressed as a product of two Fibonacci polynomials. For example,

$$h_{5,2}^{(x)} = f_4(x)f_2(x) = (x^3 + 2x)x = x^4 + 2x^2$$

and

$$h_{7,3}^{(x)} = f_5(x)f_3(x) = (x^4 + 3x^2 + 1)(x^2 + 1) = x^6 + 4x^4 + 4x^2 + 1.$$

Since $h_{m,n}^{(x)} = h_{m,m-n+1}^{(x)}$, it follows from Equation (9) that

$$h_{m,n}^{(x)} = h_{m,m-n+1}^{(x)} = f_{m-n+1}(x)f_n(x).$$

Let $m = 2r - 1$ and $n = r$. Equality (9) yields

$$h_{2r-1,r}^{(x)} = f_r(x)f_r(x) = f_r^2(x).$$

Thus, $h_{2r-1,r}^{(x)}$ is the square of a Fibonacci polynomials. In other words, the elements along the central vertical axis are Fibonacci polynomial squares. For example,

$$h_{3,2}^{(x)} = f_2^2(x) = x^2$$

and

$$h_{5,3}^{(x)} = f_3^2(x) = (x^2 + 1)^2.$$

The following result expresses the entries of the Hosoya polynomial triangle directly in terms of Lucas polynomials.

Theorem 3. *For all $m, n \geq 0$, the entries of the Hosoya polynomial triangle satisfy*

$$h_{m,n}^{(x)} = \frac{l_{m+1}(x) + (-1)^{n+1} l_{m-2n+1}(x)}{x^2 + 4}.$$

Proof. Using Equation (3), we have

$$\begin{aligned}
h_{m,n}^{(x)} &= f_{m-n+1}(x)f_n(x) \\
&= \left(\frac{\lambda^{m-n+1}(x) - \mu^{m-n+1}(x)}{\lambda(x) - \mu(x)}\right)\left(\frac{\lambda^n(x) - \mu^n(x)}{\lambda(x) - \mu(x)}\right) \\
&= \frac{\lambda^{m+1}(x) - \lambda^{m-n+1}(x)\mu^n(x) - \lambda^n(x)\mu^{m-n+1}(x) + \mu^{m+1}(x)}{x^2 + 4} \\
&= \frac{\lambda^{m+1}(x) + \mu^{m+1}(x) - (-1)^n\left(\lambda^{m-2n+1}(x) + \mu^{m-2n+1}(x)\right)}{x^2 + 4} \\
&= \frac{l_{m+1}(x) + (-1)^{n+1} l_{m-2n+1}(x)}{x^2 + 4}.
\end{aligned}$$

□

By substituting the classical Binet formulas for the Lucas polynomials into the representation above, we obtain the following Binet-type formula expression.

Corollary 4 (Binet-type formula). *Let $\lambda(x)$ and $\mu(x)$ be the roots of the characteristic polynomial $t^2 - xt - 1 = 0$. Then the entries of the Hosoya polynomial triangle admit the Binet-type formula*

$$h_{m,n}^{(x)} = \frac{\lambda^{m+1}(x) + \mu^{m+1}(x) - (-1)^n \left(\lambda^{m-2n+1}(x) + \mu^{m-2n+1}(x)\right)}{x^2+4}.$$

Using similar techniques as those in [4, 10], we can prove the following result.

Theorem 5. *The generating function for the sum of the Hosoya polynomial triangle is*

$$\sum_{m=0}^{\infty} \left(\sum_{n=0}^{m} h_{m,n}^{(x)} \right) t^m = \frac{t^2}{(1-xt-t^2)^2},$$

where the parameter x appears in the defining recurrence of the Fibonacci polynomials.

Proof. The result can be proved directly from Equations (5) and (9) by applying the techniques in Hoggatt [9]. □

3 Some identities of the Hosoya polynomial triangle

The properties of various configurations within the triangular array for the Fibonacci triangle were investigated in [10]. Here, we examine similar properties for the Hosoya polynomial triangle.

Figure 3 shows how to derive the equalities in $a) - d)$ of Proposition 6.

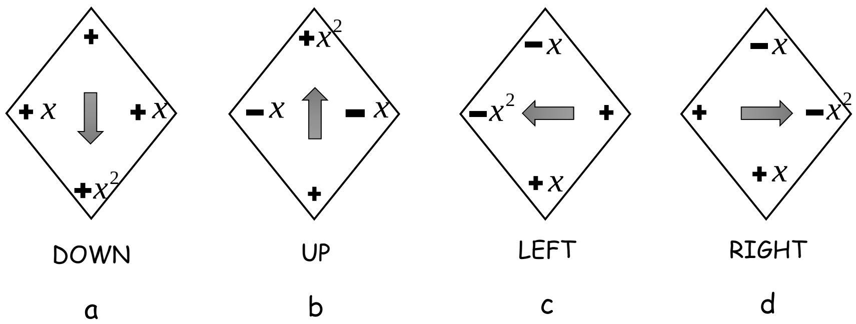

Figure 3. Diamond configuration of the Hosoya polynomial triangle

Proposition 6. *The following relations are valid:*

$a)$ $h_{m+2,n+1}^{(x)} = x^2 h_{m,n}^{(x)} + x h_{m-1,n}^{(x)} + x h_{m-1,n-1}^{(x)} + h_{m-2,n-1}^{(x)}$,

$b)$ $h_{m-4,n-2}^{(x)} = h_{m,n}^{(x)} - x h_{m-1,n}^{(x)} - x h_{m-1,n-1}^{(x)} + x^2 h_{m-2,n-1}^{(x)}$,

$c)$ $h_{m-1,n-2}^{(x)} = x h_{m,n}^{(x)} + h_{m-1,n}^{(x)} - x^2 h_{m-1,n-1}^{(x)} - x h_{m-2,n-1}^{(x)}$,

$d)$ $h_{m-1,n+1}^{(x)} = x h_{m,n}^{(x)} - x^2 h_{m-1,n}^{(x)} + h_{m-1,n-1}^{(x)} - x h_{m-2,n-1}^{(x)}$.

Proof. $a)$ Using Equation (9) and Figure 1, we obtain

$$\begin{aligned} h^{(x)}_{m+2,n+1} &= x^2 h^{(x)}_{m,n} + x h^{(x)}_{m-1,n} + x h^{(x)}_{m-1,n-1} + h^{(x)}_{m-2,n-1} \\ &= x^2 f_{m-n+1}(x) f_n(x) + x f_{m-n}(x) f_n(x) + x f_{m-n+1}(x) f_{n-1}(x) + f_{m-n}(x) f_{n-1}(x) \\ &= x f_{m-n+2}(x) f_n(x) + f_{m-n+2}(x) f_{n-1}(x) \\ &= f_{m-n+2}(x) f_{n+1}(x). \end{aligned}$$

The other cases can be proven in a manner similar to the proof of case $a)$. □

Figure 4 shows how to derive the equalities in $a)$ and $b)$ of Proposition 7.

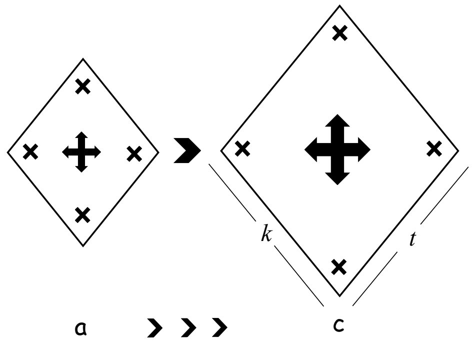

Figure 4. Multiplicative identity diamond in the Hosoya polynomial triangle

Proposition 7. *The following relations are valid:*

$a)$ $h^{(x)}_{m,n} h^{(x)}_{m-2,n-1} = h^{(x)}_{m-1,n-1} h^{(x)}_{m-1,n}$,

$b)$ $h^{(x)}_{m,n} \div h^{(x)}_{m-1,n} \div h^{(x)}_{m-1,n-1} \times h^{(x)}_{m-2,n-1} = 1$,

$c)$ $h^{(x)}_{m,n} h^{(x)}_{m-k-t,n-k} = h^{(x)}_{m-k,n-k} h^{(x)}_{m-t,n}$.

Proof. The above relations can be proven in a manner similar to the proof of case $a)$ in Proposition 6. □

The next Figure 5 shows how to derive the equalities in $b)$, $c)$ and $d)$ of Proposition 8.

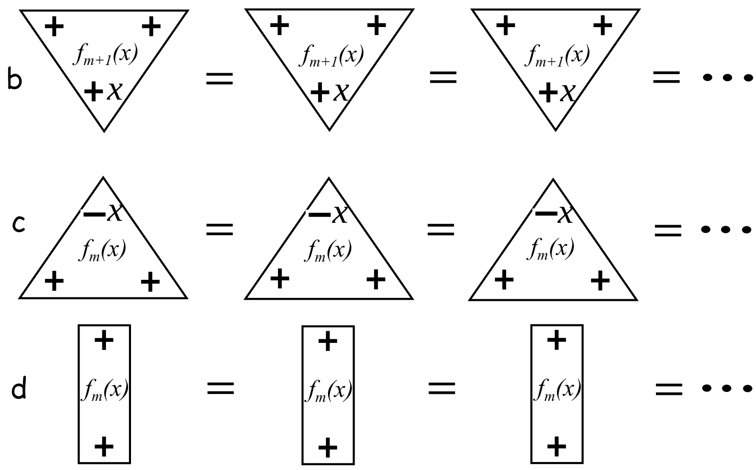

Figure 5. Summation identity triangle in the Hosoya polynomial triangle

Proposition 8. *The following relations are valid:*

$a)\ xh^{(x)}_{m,n} + h^{(x)}_{m-1,n} + h^{(x)}_{m-1,n-1} = xh^{(x)}_{m,t} + h^{(x)}_{m-1,t} + h^{(x)}_{m-1,t-1} \quad (m-1 \geq n, t \geq 1),$

$b)\ xh^{(x)}_{m,n} + h^{(x)}_{m-1,n} + h^{(x)}_{m-1,n-1} = f_{m+1}(x),$

$c)\ h^{(x)}_{m,n} + h^{(x)}_{m,n-1} - xh^{(x)}_{m-1,n-1} = f_m(x).$

$d)\ h^{(x)}_{m,n} + h^{(x)}_{m-2,n-1} = f_m(x).$

Proof. From Equations (6) and (7), we obtain

$$xh^{(x)}_{m,n} + h^{(x)}_{m-1,n} = xh^{(x)}_{m,n-1} + h^{(x)}_{m-1,n-2}. \tag{10}$$

By adding $h^{(x)}_{m-1,n-1}$ to both sides of Equation (10), we get

$$xh^{(x)}_{m,n} + h^{(x)}_{m-1,n} + h^{(x)}_{m-1,n-1} = xh^{(x)}_{m,n-1} + h^{(x)}_{m-1,n-2} + h^{(x)}_{m-1,n-1}. \tag{11}$$

If Equation (11) is transmitted along the given row m, the proof of $a)$ is completed.

Equation $b)$ can be easily derived from Equation $a)$.

Using Equations (6) and $b)$, we reach

$$h^{(x)}_{m+1,n} + h^{(x)}_{m-1,n-1} = f_{m+1}(x). \tag{12}$$

By combining Equations $b)$ and (12) with an appropriate shift of indices, we obtain Equation $c)$, and also demonstrate the transmission property related to the upward triangle (Figure 5).

Finally, multiplying Equation $b)$ by x and then processing it to Equation $c)$, we obtain Equation $d)$. □

4 Conclusion

In this paper, we have constructed and examined the Fibonacci polynomial triangle, a generalization inspired by the classical Hosoya triangle. By systematically organizing Fibonacci polynomials in a triangular array, we uncovered elegant algebraic structures and combinatorial patterns embedded within this construction.

We established a closed-form expression for the elements of the triangle, proving that each entry can be represented as the product of two Fibonacci polynomials. This identity not only simplifies the computation of entries but also reveals a symmetric property across the vertical axis of the triangle. Furthermore, we showed that elements along the central vertical line correspond to perfect squares of Fibonacci polynomials, adding a layer of structural regularity.

In addition, we derived a Binet-like formula for the triangle's entries using Lucas polynomials, enhancing the analytical tractability of the triangle. These results contribute to the growing body of knowledge on polynomial extensions of classical number sequences and demonstrate the rich interplay between recurrence relations, combinatorial identities, and algebraic representations.

Future work may involve exploring similar constructions with other polynomial sequences, such as Lucas, Pell, Jacobsthal, Padovan, Perrin or Chebyshev polynomials, or investigating applications of the Fibonacci polynomial triangle in areas such as algebraic combinatorics, coding theory, or symbolic computation.

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