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Divisibility and sequence properties of σ^+ and φ^+

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Abstract: Inspired by Lehmer's and Deaconescu's conjectures, as well as various analogue problems concerning Euler's totient function $\varphi(n)$, Schemmel's totient function $S_2(n)$, Jordan totient function J_k, and the unitary totient function $\varphi^*(n)$, we investigate analogous divisibility problems involving the functions $\sigma(n)$, $\sigma^+(n)$, and $\varphi^+(n)$. Further, we establish some interesting properties of the sequences $\{\sigma^+(n)\}_{n=1}^{\infty}$ and $\{\varphi^+(n)\}_{n=1}^{\infty}$, in particular, we prove that each of these sequences contains infinitely many arithmetic progressions of length 3.
Keywords: Euler's totient function, Unitary totient function, Schemmel's totient function, Jordan totient function, Sum of positive divisors function.
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1 Introduction

Lehmer [10] conjectured that if $\varphi(n) \mid (n-1)$, where $\varphi(n)$ is the Euler's totient function, then n must be a prime number. A positive composite integer n that satisfies the condition $\varphi(n) \mid (n-1)$ is called a Lehmer number. Let $S_2(n)$ denote Schemmel's totient function, a multiplicative function defined by

$$S_2(p^k) := \begin{cases} 0, & \text{if } p = 2 \\ p^{k-1}(p-2), & \text{if } p > 2 \end{cases},$$

where p is a prime number and $k \in \mathbb{Z}^+$. Deaconescu [5] considered Schemmel's totient function and conjectured that for $n \geq 2$

$$S_2(n) \mid \varphi(n) - 1$$

if and only if n is prime. A positive composite integer n is said to be a Deaconescu number if $S_2(n) \mid \varphi(n) - 1$. Both of these conjectures remain unproven, interesting properties of Lehmer numbers can be found in [1–3, 7, 9, 11, 12, 15], and interesting properties of Deaconescu numbers can be found in [8, 13]. Besides these problems, many analogues of Lehmer's problem have been studied, for example, the analogue of Lehmer's problem for the unitary totient function φ^*, namely the problem $\varphi^*(n) \mid (n-1)$, and for the Jordan totient function J_k, namely the problem $J_k(n) \mid (n^k - 1)$, have been investigated in [18]. The divisibility condition $(\varphi(n)+1) \mid n$ was studied in [4], and the problem $\varphi(n)^2 \mid (n^2-1)$ was examined and completely resolved in [11]. For further analogues of Lehmer's problem, interested readers may consult [6, 16]. In a recent paper, the author [14] revisits the functions

$$\varphi^+(n) \;=\; \prod_{p \mid n} (\varphi(p^{v_p(n)}) + 1), \quad \varphi^+(1) = 1$$

and

$$\sigma^+(n) \;=\; \prod_{p \mid n} (\sigma(p^{v_p(n)}) + 1), \quad \sigma^+(1) = 1$$

where $v_p(n)$ denotes the highest power of a prime number p dividing n, which were originally introduced in [17]. In this note, he investigates the behavior of

$$\sum_{\substack{n \leq x \\ \omega(n)=2}} \left(\varphi^+ - \varphi\right)(n) \quad \text{and} \quad \sum_{\substack{n \leq x \\ \omega(n)=2}} \left(\sigma^+ - \sigma\right)(n),$$

for all real $x \geq 6$. Inspired by these conjectures and problems, we investigate whether a similar statement holds for $n+1 \mid \sigma^+(n)$ and $\varphi^+(n) \mid (n-1)$. Later, we explore and establish several interesting properties of the sequences $\{\sigma^+(n)\}_{n=1}^{\infty}$ and $\{\varphi^+(n)\}_{n=1}^{\infty}$.

2 Main results

Before we begin proving results related to $\sigma^+(n)$, let us first explore some basic divisibility properties of the classical sum-of-divisors function $\sigma(n)$. Note that if $n = p$, where p is a prime, then $n+1 \mid \sigma(n)$ as $\sigma(n) = p+1$. We now attempt to determine whether there exists any positive composite integer n such that $n+1 \mid \sigma(n)$, which is equivalent to saying that there exists some positive integer k such that $\sigma(n) = k(n+1)$. It is easy to note that if $n = pq$ for some prime numbers p, q, then $n+1 \nmid \sigma(n)$ as if $n+1 \mid \sigma(n)$, then $\sigma(n) = k(n+1)$ for some positive integer k. Then $k \geq 2$ as for $k = 1$ clearly $n+1 \nmid \sigma(n)$. Note that $\sigma(pq) = k(pq+1)$ implies $\sigma(p)\sigma(q) = k(pq+1)$, since $\sigma(p) = p+1, \sigma(q) = q+1$ we have $pq+p+q+1 = kpq+k$ that is $(pq+1)(k-1) = p+q$. Since

$$\frac{p+q}{pq+1} < \frac{p+q}{pq} = \frac{1}{p} + \frac{1}{q} < 1,$$

we have

$$k = 1 + \frac{p+q}{pq+1} < 1 + 1 = 2,$$

a contradiction. Now let us consider the case $n = p^2q$, where p, q are prime numbers. Also consider $k = 2$. Since $\sigma(p^2) = 1 + p + p^2$ we have

$$\sigma(p^2q) = (1 + p + p^2)(q + 1) = 2(p^2q + 1),$$

which is equivalent to

$$q = 1 + \frac{2p}{p^2 - p - 1}. \tag{1}$$

Now choose $p = 2$ to get $q = 5$ and thus we have a desired $n = 2^2 \cdot 5 = 20$ so that $n+1 \mid \sigma(n)$.

Theorem 2.1. *If $\sigma(n) = 2(n+1)$ holds for $n = p^2q$ for some primes p, q then $p = 2, q = 5$ is the only solution.*

Proof. From (1) we get

$$q = 1 + \frac{2p}{p^2 - p - 1} = \frac{p^2 + p - 1}{p^2 - p - 1},$$

since $q \in \mathbb{N}$ we have

$$p^2 + p - 1 \equiv 0 \pmod{p^2 - p - 1},$$

that is

$$2p \equiv 0 \pmod{p^2 - p - 1}.$$

It follows that

$$p^2 - p - 1 \leq 2p$$

in other words

$$p^2 - 3p - 1 \leq 0.$$

Consider $f(x) = x^2 - 3x - 1$, then the zeros of $f(x)$ are $\frac{3 \pm \sqrt{13}}{2}$. Thus, $f(x) \leq 0$ for $x \in [\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}]$, and it follows that $p^2 - 3p - 1 \leq 0$ only for $p = 2, 3$. Now note that for $p = 3$ we have from (1) that

$$1 + \frac{6}{5} = q \notin \mathbb{N}.$$

Hence $\sigma(n) = 2(n + 1)$ holds for $n = p^2q$ when $p = 2, q = 5$. This completes the proof. □

Since we get a solution for the divisibility problem $n + 1 \mid \sigma(n)$, this naturally leads us to the following open question:

Open question 1. *Are there infinitely many positive composite integers n such that $n+1 \mid \sigma(n)$?*

Let us now consider the corresponding problem for $\sigma^+(n)$. Note that for any prime p we have $\sigma^+(p) = p + 2$, it follows that $p + 1 \nmid \sigma^+(p)$, makes the problem more interesting. Let us try to deduce a result similar to the case when $n = pq$ and $n = p^2q$, where p, q are prime numbers.

Theorem 2.2. *If $n = pq$ where p, q are prime numbers, then $n + 1 \nmid \sigma^+(n)$.*

Proof. Let $n = pq$ for some prime numbers p, q. If possible, suppose that $\sigma^+(n) = k(n+1)$ for some positive integer k. Then $k \geq 2$ as for $k = 1$ clearly $n + 1 \nmid \sigma(n)$. Note that

$$\sigma^+(pq) = k(pq + 1),$$

which implies

$$(p + 2)(q + 2) = k(pq + 1)$$

from which we can write

$$(pq + 1)(k - 1) = 2(p + q) + 3. \tag{2}$$

Since

$$\frac{2(p+q)+3}{pq+1} < \frac{2(p+q)+3}{pq} = \frac{2}{p} + \frac{2}{q} + \frac{3}{pq} \leq 1 + \frac{2}{3} + \frac{1}{2} < 3,$$

we have $k < 4$. Now for $k = 2$, from (2) we get

$$pq + 1 = 2p + 2q + 3, \tag{3}$$

which implies that $2 \mid pq$. Without loss of generality, set $p = 2$, then from (3) we get $6 = 0$, a contradiction. Now for $k = 3$, we get from (2) that $2 \mid 2p + 2q + 3$ implies $2 \mid 3$, again a contradiction. Therefore, for any positive integer k, $\sigma^+(n) = k(n + 1)$ does not hold. This completes the proof. □

Theorem 2.3. *For any $n = p^2q$ where p, q are prime numbers, $\sigma^+(n) = 2(n + 1)$ does not hold.*

Proof. Note that

$$\sigma^+(n) = \sigma^+(p^2)\sigma^+(q) = (2 + p + p^2)(2 + q) = 2(p^2q + 1)$$

implies that

$$q = \frac{2(p^2 + p + 1)}{p^2 - p - 2} = 2 + \frac{4p + 6}{p^2 - p - 2}. \tag{4}$$

Also observe that $q - 2 \in \mathbb{N}$ as if $q - 2 = 0$, i.e., $q = 2$, then from (4) we get $p = -\frac{3}{2}$. It follows that $p^2 - p - 2 \mid 4p + 6$, therefore

$$p^2 - p - 2 \leq 4p + 6$$

in other words

$$p^2 - 5p - 8 \leq 0.$$

Consider $f(x) = x^2 - 5x - 8$, then the zeros of $f(x)$ are $\frac{5 \pm \sqrt{57}}{2}$. Thus, $f(x) \leq 0$ for all $x \in [\frac{5-\sqrt{57}}{2}, \frac{5+\sqrt{57}}{2}]$, it follows that $f(p) = p^2 - 5p - 8 \leq 0$ only for $p = 2, 3, 5$. Now note that p cannot be 2 from (4). Also, for $p = 3, 5$ we get from (4) that $q = 13/2, 31/9 \notin \mathbb{N}$, respectively. Thus, for any prime numbers p, q, $2(p^2q + 1) = \sigma^+(p^2q)$ does not hold. This completes the proof. □

Theorem 2.4. *If $n = p_1^{r_1} \cdots p_r^{r_r}$, where p_j are prime numbers, and $r_j \geq 1$, then*

$$\frac{1}{2}I(n) < \frac{\sigma^+(n)}{n+1} < 2^r I(n),$$

where $I(n)$ is the abundancy index of n.

Proof. Let $n = p_1^{r_1} \cdots p_r^{r_r}$, where p_j are prime numbers, and $r_j \geq 1$ then

$$\frac{1}{2}I(n) = \frac{\sigma(n)}{2n} < \frac{\sigma(n)}{n+1} < \frac{\sigma^+(n)}{n+1}.$$

Also, since

$$\sigma^+(n) = \prod_{j=1}^{r}\left(\sigma(p_j^{r_j}) + 1\right) < \prod_{j=1}^{r}\left(\sigma(p_j^{r_j}) + \sigma(p_j^{r_j})\right) = 2^r \prod_{j=1}^{r} \sigma(p_j^{r_j}) = 2^r \sigma(n),$$

we have

$$\frac{\sigma^+(n)}{n+1} < \frac{\sigma^+(n)}{n} < \frac{2^r\sigma(n)}{n} = 2^r I(n).$$

This completes the proof. □

We have checked all positive composite integers n from 1 to 10^5 but none of them satisfied the condition that $\sigma^+(n)$ is divisible by $n+1$. Therefore, we are offering some open questions.

Open question 2. *Is there any positive composite integer n such that $n + 1 \mid \sigma^+(n)$?*

Open question 3. *Are there infinitely many positive composite integers n such that $n+1 \mid \sigma^+(n)$?*

We now establish some interesting properties of the sequence $\{\sigma^+(n)\}_{n=1}^{\infty}$.

Theorem 2.5. *In the sequence $\{\sigma^+(n)\}_{n=1}^{\infty}$, there exist infinitely many pairs $(n, m) \in \mathbb{N}^2$ with $n \neq m$ such that $\sigma^+(n) = \sigma^+(m)$.*

Proof. Let $n = 2^k \cdot 5$ and $m = 2^{k-1} \cdot 9$, where $k \geq 2$, then $\sigma^+(n) = \sigma^+(2^k) \cdot \sigma^+(5) = 2^{k+1} \cdot 7 = 2^k \cdot 14 = \sigma^+(2^{k-1}) \cdot \sigma^+(9) = \sigma^+(2^{k-1} \cdot 9) = \sigma^+(m)$. Since the number of such values of k is infinite, so are the corresponding pairs (n, m) satisfying $\sigma^+(n) = \sigma^+(m)$. □

Theorem 2.6. *The sequence $\{\sigma^+(n)\}_{n=1}^{\infty}$ contains infinitely many arithmetic progressions of length 3.*

Proof. Let $a = 2^k$, $b = 2^{k+2}$ and $c = 2^{k+1} \cdot 5$, where $k \geq 1$. To prove the theorem, it suffices to show that $2\sigma^+(b) = \sigma^+(a) + \sigma^+(c)$. The infinitude of arithmetic progressions of length 3 then follows from the infinitude of such values of k. We can see that

$$2\sigma^+(b) = 2^{k+4} = 2^{k+1} + 2^{k+1} \cdot 7 = \sigma^+(2^k) + \sigma^+(2^k \cdot 5) = \sigma^+(a) + \sigma^+(c).$$

This completes the proof. □

Inspired by the above theorem, we propose the following conjecture:

Conjecture 2.1. *For each prime $q \geq 3$, there exist infinitely many pairs (p, n) such that $\sigma^+(n) = 2p - q$, where p is a prime number.*

We now proceed to derive analogous results for the function $\varphi^+(n)$. Note that if $n = p$, where p is a prime, then $\varphi^+(n) \nmid (n-1)$ as $\varphi^+(n) = \varphi(n) + 1 = (p-1) + 1 = p$. We now attempt to determine whether there exists any positive composite integer n such that $\varphi^+(n) \mid (n-1)$, which is equivalent to saying that there exists some positive integer k such that $(n-1) = k\varphi^+(n)$.

Theorem 2.7. *If $n = p_1p_2\cdots p_r$, where p_i are prime numbers, then $\varphi^+(n) \nmid (n-1)$.*

Proof. Let $n = p_1p_2\cdots p_r$, where p_i are prime numbers. If possible, suppose that $(n-1) = k\varphi^+(n)$ for some positive integer k. Then

$$k\varphi^+(n) = kp_1p_2\cdots p_r = p_1p_2\cdots p_r - 1,$$

that is

$$k = \frac{p_1p_2\cdots p_r - 1}{p_1p_2\cdots p_r} < 1,$$

a contradiction. Therefore $\varphi^+(n) \nmid (n-1)$. □

Let us consider the case $n = p^2$, where p is a prime number. Also consider $k = 1$. Since $\varphi^+(p^2) = p^2 - p + 1$, we have

$$p^2 - 1 = p^2 - p + 1,$$

which gives $p = 2$, and thus we get a positive composite integer $n = 4$ such that $\varphi^+(n) \mid (n-1)$. Now we prove that if $n = p^r$ for which $\varphi^+(n) \mid (n-1)$ then $r = 2$ and $p = 2$.

Theorem 2.8. *If $\varphi^+(n) \mid (n-1)$, where $n = p^r$, p is a prime number, then $r = 2$ and $p = 2$.*

Proof. Let $p^r - 1 = k\varphi^+(p^r)$ holds for some positive integer k. If possible, suppose that $k \geq 2$. Then

$$p^r - 1 = k\varphi^+(p^r) = k(p^r - p^{r-1} + 1),$$

which is equivalent to

$$k = \frac{p^r - 1}{p^r - p^{r-1} + 1} < \frac{p^r}{p^{r-1}(p-1)} = \frac{p}{p-1} \leq 2,$$

that is $k < 2$, which is a contradiction. Therefore, we must have $k = 1$, which implies that $p^r - 1 = \varphi^+(p^r) = p^r - p^{r-1} + 1$, which implies that $p^{r-1} = 2$, which is true when $r = 2$ and $p = 2$. This completes the proof. □

Theorem 2.9. *If $n = p_1^{r_1}\cdots p_r^{r_r}$, where p_j are prime numbers, and $r_j \geq 2$, then*

$$\prod_{j=1}^{r}\left(1 + \frac{1}{p_j^2}\right) \leq \frac{n-1}{\varphi^+(n)} < 2^r.$$

Proof. We first prove that, for each p_j the following holds

$$\left(1 + \frac{1}{p_j^2}\right)(p_j^{r_j} - p_j^{r_j-1} + 1) < p_j^{r_j},$$

which is equivalent to prove that

$$p_j^{r_j+2} - (1+p_j^2)(p_j^{r_j} - p_j^{r_j-1} + 1) > 0.$$

Note that for $p_j \geq 2$ and $r_j \geq 2$, we have

$$p_j^{r_j+2} - (1+p_j^2)(p_j^{r_j} - p_j^{r_j-1} + 1) = p_j^{r_j}(p_j - 1) + p_j^{r_j-1} - p_j^2 - 1 > 0.$$

Therefore

$$\prod_{j=1}^{r}\left(1+\frac{1}{p_j^2}\right)(p_j^{r_j} - p_j^{r_j-1} + 1) < \prod_{j=1}^{r} p_j^{r_j},$$

that is

$$\prod_{j=1}^{r}\left(1+\frac{1}{p_j^2}\right)(p_j^{r_j} - p_j^{r_j-1} + 1) \leq \prod_{j=1}^{r} p_j^{r_j} - 1 = n - 1,$$

since $\prod_{j=1}^{r}(p_j^{r_j} - p_j^{r_j-1} + 1) = \varphi^+(n)$, we have

$$\prod_{j=1}^{r}\left(1+\frac{1}{p_j^2}\right) \leq \frac{n-1}{\varphi^+(n)}.$$

Also, note that, for each p_j, we have

$$\frac{p_j^{r_j}}{p_j^{r_j} - p_j^{r_j-1}} = \frac{p_j}{p_j - 1} \leq 2,$$

then

$$\frac{n-1}{\varphi^+(n)} < \frac{n}{\varphi^+(n)} < \prod_{j=1}^{r} \frac{p_j^{r_j}}{p_j^{r_j} - p_j^{r_j-1}} \leq 2^r.$$

This completes the proof. □

We have verified, through exhaustive computation, that there is only one positive integer $n \leq 10^5$ such that

$$\varphi^+(n) \mid (n-1).$$

Based on this empirical observation, we propose the following two open questions:

Open question 4. *Is there any positive composite integer $n \neq 4$ such that $\varphi^+(n) \mid (n-1)$?*

Open question 5. *Are there infinitely many positive composite integers n such that $\varphi^+(n) \mid (n-1)$?*

We now establish similar interesting properties for the sequence $\{\varphi^+(n)\}_{n=1}^{\infty}$.

Theorem 2.10. *In the sequence $\{\varphi^+(n)\}_{n=1}^{\infty}$, there exist infinitely many pairs $(n, m) \in \mathbb{N}^2$ with $n \neq m$ such that $\varphi^+(n) = \varphi^+(m)$.*

Proof. Let $n = 7p$ and $m = 9p$, where p is a prime other than $3, 7$, then $\varphi^+(n) = 7p = (6+1)p = (\varphi(3^2)+1)p = \varphi^+(9)p = \varphi^+(9p) = \varphi^+(m)$. Since the number of prime p is infinite, so are the corresponding pairs (n, m) satisfying $\varphi^+(n) = \varphi^+(m)$. □

Theorem 2.11. *The sequence* $\{\varphi^+(n)\}_{n=1}^{\infty}$ *contains infinitely many arithmetic progressions of length* 3*.*

Proof. Let $a = 3p$, $b = 7p$ and $c = 11p$, where p is a prime number greater than or equal to 13. To prove the theorem, it suffices to show that $2\varphi^+(b) = \varphi^+(a) + \varphi^+(c)$. The infinitude of arithmetic progressions of length 3 then follows from the infinitude of prime p. We can observe that

$$2\varphi^+(b) = 14p = 3p + 11p = \varphi^+(a) + \varphi^+(c),$$

which completes the proof. □

We have a conjecture that, for infinitely many primes p, $\varphi(p+1) > \varphi(p)$, this conjecture is still open. We give a similar result for $\varphi^+(n)$.

Theorem 2.12. *For all primes* $p \in$ OEIS A005382 *and* $p > 2$*, we have* $\varphi^+(q) < \varphi^+(q+1)$*, where* $q = 2p - 1$ *is a prime.*

Proof. Since $q = 2p - 1$ is a prime, we have $\varphi^+(q) = 2p - 1$ and $\varphi^+(q+1) = \varphi^+(2p) = 2p$, thus $\varphi^+(q) < \varphi^+(q+1)$. □

Open question 6. *Are there infinitely many prime* p *such that* $\varphi^+(p) < \varphi^+(p+1)$*?*

We end this paper with an analogous conjecture to Conjecture 1.

Conjecture 2.2. *For each prime* $q \geq 3$*, there exist infinitely many pairs* (p, n)*, such that* $\varphi^+(n) = 2p - q$*, where* p *is a prime number.*

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