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On b-repdigit polygonal numbers

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Abstract: We prove a finiteness theorem concerning repdigits in base $b \geq 2$ represented by a fixed quadratic polynomial. We also show that there is a finite number of polygonal numbers that are also b-repdigits for all $b \geq 2$ provided that $(b, s) \notin \left\{ \left(\frac{8(s-2)}{(s-4)^2} d + 1, s \right) : s \in [3, 13] - \{4\} \right\}$, where $s \geq 3$ denotes the number of sides of the polygon and $d \in \{1, 2, \ldots, b-1\}$. We illustrate this result by finding all triangular, pentagonal and heptagonal numbers that are also b-repdigits for $b \in [2, 9]$. This paper is motivated by a previous work of Kafle, Luca, and Togbé who considered the same finiteness problem for $b = 10$ to find all pentagonal and heptagonal numbers that are also repdigits.

Keywords: Repdigit, Polygonal number, Bachet equation, Mordell curve.
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1 Introduction

A *repdigit* is a number composed of repetition of a single digit. Repdigits are of the form

$$d\left(\frac{10^m - 1}{9}\right),$$

for some $m \geq 1$ and $d \in \{1, 2, \ldots, 9\}$. If the digits of a repdigit are all 1's, it is known as a *repunit*. The term "repunit" was coined by Beiler [2, Chapter "11111 . . . 111"]. A study of prime factorizations, relationships with the periodicity of decimals, and other aspects of repunits can be found in Yates [11].

Repdigits can be generalized to base $b \geq 2$, giving a *b-repdigit* as number of the form

$$d\left(\frac{b^m - 1}{b - 1}\right),$$

for some $m \geq 1$ and $d \in \{1, 2, \ldots, b-1\}$. We denote a b-repdigit number by $R_d(b, m)$. If $d = 1$, we obtain *b-repunit numbers*, which we denote by $R_m^{(b)}$. In particular, if $b = 2$, we obtain the 2-repdigits numbers (or *Mersenne numbers*) $R_m^{(2)} := M_m$. The following Table 1 gives the first few b-repdigits and b-repunits in bases $b = 2$ to 10 with their respective entries in the On-Line Encyclopedia of Integer Sequences (OEIS) [10].

Table 1. First b-repdigits and b-repunits for $b \in [2, 10]$

b	**OEIS**	**b-repdigits**	**OEIS**	**b-repunits**
2	A000225	$1, 3, 7, 15, 31, 63, 127 \ldots$	A000225	$1, 3, 7, 15, 31, 63, 127 \ldots$
3	A048328	$1, 2, 4, 8, 13, 26, 40, 80, 121, \ldots$	A003462	$1, 4, 13, 40, 121, 364, \ldots$
4	A048329	$1, 2, 3, 5, 10, 15, 21, 42, 63, \ldots$	A002450	$1, 5, 21, 85, 341, 1365, \ldots$
5	A048330	$1, 2, 3, 4, 6, 12, 18, 24, 31, 62, \ldots$	A003463	$1, 6, 31, 156, 781, 3906, \ldots$
6	A048331	$1, 2, 3, 4, 5, 7, 14, 21, 28, 35, \ldots$	A003464	$1, 7, 43, 259, 1555, 9331, \ldots$
7	A048332	$1, 2, 3, 4, 5, 6, 8, 16, 24, 32, \ldots$	A023000	$1, 8, 57, 400, 2801, 19608, \ldots$
8	A048333	$1, 2, 3, 4, 5, 6, 7, 9, 18, 27, 36, \ldots$	A023001	$1, 9, 73, 585, 4681, 37449, \ldots$
9	A048334	$1, 2, 3, 4, 5, 6, 7, 8, 10, 20, 30, \ldots$	A002452	$1, 10, 91, 820, 7381, 66430, \ldots$
10	A010785	$1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 22, \ldots$	A002275	$1, 11, 111, 1111, 11111, \ldots$

The n-th s-sided polygonal number, denoted by P_n^s, is the number of counters that can be arranged into a s-sided polygon with n counters along each side. A formula for P_n^s is

$$\frac{(s-2)n^2 - (s-4)n}{2},$$

for positive integers $n \geq 2$ and $s \geq 3$. By convention, $P_1^s = 1$ is the first polygonal number for any $s \geq 3$. The Table 2 illustrates the first polygonal numbers for a given value of sides.

Table 2. First polygonal numbers for $s \in [3, 7]$

s	**Name**	**OEIS**	**A first few terms**
3	Triangular numbers	A000217	$1, 3, 6, 10, 15, 21, 28, 36, \ldots$
4	Square numbers	A000290	$1, 4, 9, 16, 25, 36, 49, 64, \ldots$
5	Pentagonal numbers	A000326	$1, 5, 12, 22, 35, 51, 70, \ldots$
6	Hexagonal numbers	A000384	$1, 6, 15, 28, 45, 66, 91, \ldots$
7	Heptagonal numbers	A000566	$1, 7, 18, 34, 55, 81, 112, \ldots$

2 Main results

In 2020, Kafle, Luca, and Togbé [6, 7] proved the finiteness of the solutions of some equations involving repdigits. As a consequence, they identified the pentagonal and heptagonal numbers which are also repdigits. Inspired by their result, we consider the same finiteness problem with b-repdigits instead of repdigits. Our main result is the following.

Theorem 2.1. *Let A, B, C be fixed rational numbers with $A \neq 0$. Then the Diophantine equation*

$$An^2 + Bn + C = d\left(\frac{b^m - 1}{b - 1}\right) \tag{1}$$

has only a finite number of solutions, in integers $m, n \geq 1$, $b \geq 2$, and $d \in \{1, 2, \ldots, b-1\}$ provided that $(b-1)(B^2 - 4AC) - 4Ad \neq 0$.

Proof. We start by multiplying both sides of equation (1) by $4A$. Then, we add B^2 to both sides of the resulting equation, which gives us

$$(2An + B)^2 = \frac{4Ab^m d}{b-1} + B^2 - 4AC - \frac{4Ad}{b-1}.$$

Writing $m = 3m_1 + r$ with $m_1 \geq 0$ and $r \in \{0, 1, 2\}$, and multiplying both sides of the above equation by $16A^2 b^{2r}(b-1)^3 d^2$, we get to rewrite it as

$$y^2 = x^3 + k, \tag{2}$$

where

$$y := 4Ab^r d(b-1)^2(2An + B), \quad x := 4Ab^{m_1+r} d(b-1),$$

and

$$k := 16A^2 b^{2r} d^2 (b-1)^3 \left[(b-1)(B^2 - 4AC) - 4Ad\right].$$

Since $A \neq 0$, $b \geq 2$, $d \geq 1$ and $(b-1)(B^2-4AC)-4Ad \neq 0$, we have that $k \neq 0$. Thus, we obtain an elliptic curve over $\mathbb{Q}$ given by (2). By a Siegel's Theorem of 1926 (see Mollin, [9, p. 313]), Equation (2) has only finitely many solutions $x, y \in \mathbb{Z}$ since $27k^2 \neq 0$. As a consequence, Equation (1) has only a finite number of positive integer solutions. This completes the proof of Theorem 2.1. □

Now, we state some additional applications of Theorem 2.1. In 1998, Keith [8] considered the problem of determining which polygonal numbers are repdigits. The converse is easy—every repdigit number is trivially a polygonal number, because every integer n is equal to both P_n^2 and P_2^n. He presented an efficient algorithm for finding repdigit polygonal numbers and used it to provide a complete characterization of all 1526 such numbers with 50 or fewer digits.

We consider Keith's problem with b-repdigit numbers instead of repdigit numbers and prove the following as a consequence of Theorem 2.1.

Corollary 2.1. *Let $s \geq 3$ be a fixed integer. Then the Diophantine equation*

$$P_n^s = d\left(\frac{b^m - 1}{b - 1}\right) \tag{3}$$

has only a finite number of solutions, in integers $m, n \geq 1$, $b \geq 2$, and $d \in \{1, 2, \ldots, b-1\}$ whenever $(b, s) \notin \{(8d+1, 3), (24d+1, 5), (8d+1, 6), ((40d/9)+1, 7), (3d+1, 8), ((56d/25)+1, 9), ((16d/9)+1, 10), ((72d/49)+1, 11), ((5d/4)+1, 12), ((88d/81)+1, 13)\}$.

Proof. Setting $A = (s-2)/2$, $B = -(s-4)/2$ and $C = 0$ in Theorem 2.1 we can conclude that Equation (3) has finite solutions if $(b-1)(s-4)^2 - 8d(s-2) \neq 0$.

Otherwise, $d = (b-1)(s-4)^2/8(s-2) \leq b-1$ implies that $s^2 - 16s + 32 \leq 0$. The solution set for this quadratic inequality is $[8-4\sqrt{2}, 8+4\sqrt{2}]$. Since $s \geq 3$ and $d \geq 1$, we obtain that $s \in [3,13] - \{4\}$. Therefore, we can conclude that Equation (3) has finite solutions if

$$(b,s) \notin \left\{ \left(\frac{8(s-2)}{(s-4)^2} d + 1, s \right) : s \in [3,13] - \{4\} \right\}. \qquad \square$$

With $b = 10$ in Corollary 2.1, we can reach the following conclusion.

Corollary 2.2. *If $(d,s) \neq (3,8)$, then there are only a finite number of polygonal numbers that are repdigits. In the case $(d,s) = (3,8)$, there are infinitely many polygonal numbers that are repdigits, which can be given explicitly.*

Proof. In the case $(d,s) = (3,8)$, the equation $3n^2 - 2n = (10^m - 1)/3$ has infinitely many solutions given by $(m,n) = (2t, (1-10^t)/3)$ for all $t \in \mathbb{Z}^+$. $\square$

If $d = 1$ in Corollary 2.1 above, we have the following result.

Corollary 2.3. *There are only a finite number of polygonal numbers that are also b-repunit numbers for all $b \geq 2$ whenever $(b,s) \notin \{(9,3),(25,5),(9,6),(4,8)\}$. In these cases, there are infinitely many that can be given explicitly.*

Proof. For particular cases, we have the following infinite solutions.

Table 3. Infinite polygonal b-repunits.

(b,s)	Diophantine equation	Solutions
$(9,3)$	$n(n+1) = (9^m-1)/4$	$n = (3^m-1)/2$ for $m \geq 1$
$(25,5)$	$n(3n-1) = (25^m-1)/12$	$n = (5^m+1)/6$ for $m \geq 1$ odd
$(9,6)$	$n(2n-1) = (9^m-1)/8$	$n = (3^m+1)/4$ for $m \geq 1$ odd
$(4,8)$	$n(3n-2) = (4^m-1)/3$	$n = (2^m+1)/3$ for $m \geq 1$ odd

$\square$

Next, we identify some b-repdigit polygonal numbers for b and s fixed. First, note that

$$P_n^s = R_{P_n^s}(b,1) \quad \text{for all} \quad b \geq 2,\ n \geq 1,\ \text{and } s \geq 3.$$

In addition,

$$P_n^s = R_2^{(P_n^s - 1)} \quad \text{for all } n \geq 1 \text{ and } s \geq 3.$$

Also,

$$P_{2n+1}^s = R_{2n+1}((s-2)n, 2) \quad \text{for all } n \geq 2 \text{ and } s \geq 5.$$

Moreover,

$$P_n^s = \begin{cases} R_2(\frac{P_n^s}{2} - 1, 2) & \text{if } s \equiv 0,2 \pmod 4 \text{ and } n \equiv 0 \pmod 2; \\ R_2(\frac{P_n^s}{2} - 1, 2) & \text{if } s \equiv 1,3 \pmod 4 \text{ and } n \equiv 0 \pmod 4; \\ R_2(\frac{P_n^s}{2} - 1, 2) & \text{if } s \equiv 1,3 \pmod 4 \text{ and } n \equiv 3 \pmod 4). \end{cases}$$

We call these solutions, *trivial b-repdigit polygonal numbers*.

2.1 b-repdigit triangular numbers

According to Ballew and Weger [1], Escott proved in 1905 that $1, 3, 6, 55, 66, 666$ are the only triangular numbers of less than 30 digits that are repdigits. In 1975, they [1] proved that, in fact, these are the only repdigit triangular numbers.

In this section, we generalize the Ballew and Weger problem by considering b-repdigits instead of repdigits. We denote the n-th triangular number P_n^3 by T_n. First, note that $T_{2n} = R_n(2n, 2)$ for all $n \geq 1$. In addition, $T_{2n+1} = R_{n+1}(2n, 2)$ for all $n \geq 2$. Moreover, $T_{6T_n} = R_{T_n}(3n + 1, 3)$ for all $n \geq 1$. We refer to these solutions as *trivial b-repdigit triangular numbers*. Our result is as follows.

Corollary 2.4. *There are only a finite number of triangular numbers which are also b-repdigit numbers for all $b \geq 2$, except when $b = 8d + 1$ where there are infinitely many. The only non-trivial b-repdigit triangular numbers for $b \in [2, 9]$ are:*

$$\begin{aligned} T_5 &= 15 = M_4; \\ T_{90} &= 4095 = M_{12} = R_3(4, 6) = R_7(8, 4). \end{aligned}$$

Proof. From Corollary 2.1 with $s = 3$ we conclude that Diophantine equation

$$\frac{n(n+1)}{2} = d\left(\frac{b^m - 1}{b - 1}\right) \tag{4}$$

has only a finite number of solutions, in integers $m, n \geq 1$, $b \geq 2$ and $d \in \{1, 2, \ldots, b - 1\}$ provided that $b \neq 8d + 1$. In case $b = 8d + 1$, Equation (4) have infinitely many solutions. If d is a triangular number, then $n = ((8d + 1)^{m/2} - 1)/2$ for all $m \geq 1$ is a solution that satisfies (4) identically. If d is not a triangular number, then $n = ((8d + 1)^{m/2} - 1)/2$ for all even $m \geq 1$ is a solution satisfying (4) identically.

As in the proof of Theorem 2.1 we can transform Equation (4) into Bachet's equation

$$y^2 = x^3 + k, \tag{5}$$

where now

$$y := b^r d(b-1)^2(2n+1), \quad x := 2b^{m_1+r} d(b-1),$$

and

$$k := b^{2r} d^2 (b-1)^3 (b - 8d - 1).$$

We note that k is nonzero. Since $b \in [2, 9]$, $d \in \{1, \ldots, b - 1\}$, $b \neq 8d + 1$, and $r \in \{0, 1, 2\}$ we obtain one hundred and five Mordell curves given by (5). Now, we use *MAGMA* [3] to determine the integer points (x, y) on each of these elliptic curves.

The elliptic curve (5) has no integer points for $(b, d, r) = (3, 1, 2)$, $(3, 2, 2)$, $(4, 1, 2)$, $(4, 2, 1)$, $(5, 2, 1)$, $(5, 2, 2)$, $(5, 4, 2)$, $(6, 2, 1)$, $(6, 2, 2)$, $(6, 4, 1)$, $(7, 2, 1)$, $(7, 2, 2)$, $(7, 4, 1)$, $(7, 4, 2)$, $(7, 5, 1)$, $(7, 5, 2)$, $(7, 6, 2)$, $(9, 2, 2)$, $(9, 3, 2)$, $(9, 4, 1)$, $(9, 5, 1)$, $(9, 6, 2)$, $(9, 8, 1)$.

While for $(b, d, r) = (3, 1, 0)$, $(3, 2, 0)$, $(3, 2, 1)$, $(4, 2, 0)$, $(5, 1, 0)$, $(5, 2, 0)$, $(5, 3, 0)$, $(5, 3, 2)$, $(5, 4, 0)$, $(5, 4, 1)$, $(6, 1, 0)$, $(6, 1, 2)$, $(6, 2, 0)$, $(6, 3, 0)$, $(6, 4, 0)$, $(6, 5, 0)$, $(6, 5, 1)$, $(6, 5, 2)$, $(7, 1, 0)$,

$(7,1,2)$, $(7,2,0)$, $(7,3,2)$, $(7,4,0)$, $(7,5,0)$, $(7,6,0)$, $(8,2,0)$, $(8,2,1)$, $(8,2,2)$, $(9,2,0)$, $(9,2,1)$, $(9,3,0)$, $(9,4,0)$, $(9,4,2)$, $(9,5,0)$, $(9,5,2)$, $(9,6,0)$, $(9,7,0)$, $(9,7,1)$, $(9,7,2)$, $(9,8,0)$, $(9,8,2)$ the elliptic curve (5) has integer points (x, y) but they do not give rise to positive integers m, n.

The following Table 4 displays all the integer points (x, y) as described above that produce the corresponding non-trivial integer solutions of Equation (4) for $b \in [2, 9]$. The other elliptic curves have integer points (x, y) that produce trivial solutions of equation (4), so we omit them. The (x, y)'s in **bold** correspond to the integers m, n in the sixth column. For instance, $(b, d, r) = (2, 1, 0)$ implies that $x = 2^{m_1+1}$ and $y = 2n + 1$. If $(x, y) = (32, 181)$, then $m_1 = 4$ (and thus $m = 12$) and $n = 90$.

Table 4. Integer solutions (x, y).

$\boldsymbol{b}$	$\boldsymbol{d}$	$\boldsymbol{r}$	$\boldsymbol{k}$	$\boldsymbol{(x, y)}$	$\boldsymbol{(m, n)}$
2	1	0	-7	$(2, \pm 1), (\mathbf{32}, \mathbf{181}), (32, -181)$	$(12, 90)$
2	1	1	-28	$(\mathbf{4}, \mathbf{6}), (4, -6), (8, \pm 22), (37, \pm 225)$	$(4, 5)$
4	3	0	-5103	$(18, \pm 27), (\mathbf{288}, \mathbf{4887}), (288, -4887)$	$(6, 90)$
8	7	0	-823543	$(98, \pm 343), (\mathbf{1568}, \mathbf{62083}), (1568, -62083)$	$(4, 90)$
8	7	1	-52706752	$(392, \pm 2744), (\mathbf{6272}, \mathbf{496664}), (6272, -496664)$	$(4, 90)$
8	7	2	-3373232128	$(1568, \pm 21952), (\mathbf{25088}, \mathbf{3973312}),$ $(25088, -3973312)$	$(4, 90)$

The list of ordered pairs (m, n) in the sixth column of Table 4 above, together with the corresponding values of b and d in the first and second columns, give us the complete list of non-trivial solutions (b, d, m, n) for Equation (4). From this, we can deduce that the only non-trivial triangular numbers in the sequence of b-repdigits for $b \in [2, 9]$ are given by the statement in Corollary 2.4. This concludes the proof of Corollary 2.4. □

In 2010, Jaroma [4] proved that 1 is the only integer that is both triangular and repunit. We have shown the following.

Corollary 2.5. *There are only a finite number of integers that are both triangular and b-repunits for all $b \geq 2$, except when $b = 9$ where there are infinitely many given by $T_{(3^m-1)/2} = R_m^{(9)}$ for all $m \geq 1$. The only non-trivial triangular numbers that are b-repunits for $b \in [2, 8]$ are* 15 *and* 4095.

2.2 b-repdigit pentagonal numbers

In 2020, Kafle, Luca, and Togbé [5,6] studied the pentagonal numbers $(P_n)_n$ found in the repdigit sequence showing that the only ones are 1, 5, and 22. In this section, we extend their result by studying the pentagonal numbers present in the b-repdigit sequence. Our result is the following.

Corollary 2.6. *There are only a finite number of pentagonal numbers that are also b-repdigit numbers for all $b \geq 2$, except when $b = 24d + 1$ where there are infinitely many. There are no non-trivial b-repdigit pentagonal numbers for $b \in [2, 9]$.*

Proof. Let $s = 5$ in Corollary 2.1. Then the Diophantine equation

$$\frac{n(3n-1)}{2} = d\left(\frac{b^m - 1}{b-1}\right) \tag{6}$$

has only a finite number of solutions, in integers $m, n \geq 1$, $b \geq 2$ and $d \in \{1, 2, \ldots, b-1\}$ whenever $b \neq 24d + 1$. In case $b = 24d + 1$, Equation (6) have infinitely many solutions. If d is a pentagonal number, then $n = (1 + (24d + 1)^{m/2})/6$ for odd $m \geq 1$ is a solution that holds (6) identically.

Now, working as in the proof of Theorem 2.1, Equation (6) can be written as

$$y^2 = x^3 + k, \tag{7}$$

where this time

$$y := 3b^r d(b-1)^2(6n-1), \quad x := 6b^{m_1+r}d(b-1),$$

and

$$k := 9b^{2r}d^2(b-1)^3(b - 24d - 1).$$

Note that $k \neq 0$. Since $b \in [2, 9]$, $d \in \{1, \ldots, b-1\}$, and $r \in \{0, 1, 2\}$ we obtain 108 Mordell curves given by (7). This leads us to determine the integer points (x, y) on each of these elliptic curves. To do this, we use *MAGMA* [3]. Of the 108 elliptic curves:

- 13 do not have integer points, namely in $(b, d, r) = (3, 1, 2)$, $(4, 2, 2)$, $(5, 3, 1)$, $(6, 2, 2)$, $(6, 3, 1)$, $(7, 2, 2)$, $(7, 4, 2)$, $(7, 6, 1)$, $(7, 6, 2)$, $(9, 1, 2)$, $(9, 2, 2)$, $(9, 4, 1)$, $(9, 6, 2)$,
- 82 have integer points (x, y) but do not give rise to positive integers $m, n \geq 1$, and
- 13 have integer points that give rise to trivial solutions of Equation (6), namely in $(b, d, r) =$ $(2, 1, 1)$, $(3, 1, 1)$, $(4, 1, 1)$, $(5, 1, 1)$, $(6, 1, 1)$, $(7, 1, 1)$, $(8, 1, 1)$, $(9, 1, 1)$, $(4, 1, 2)$, $(6, 5, 1)$, $(7, 5, 1)$, $(8, 5, 1)$, $(9, 5, 1)$.

This completes the proof of Corollary 2.6. □

2.3 *b*-repdigit heptagonal numbers

Recently, Kafle, Luca, and Togbé [5, 6] proved that 1, 7, and 55 are the only heptagonal numbers $(H_n)_n$ that are repdigits. We prove the following about heptagonal numbers that are b-repdigits.

Corollary 2.7. *Let b, d, m, n be positive integers such that $b \geq 2$ and $d \in \{1, 2, \ldots, b-1\}$. Then the Diophantine equation*

$$H_n = R_d(b, m) \tag{8}$$

has only a finite number of solutions whenever $b \neq (40d/9) + 1$. Otherwise, Equation (8) has infinitely many solutions. The only non-trivial b-repdigit heptagonal numbers for $b \in [2, 9]$ are:

$$H_2 = 7 = M_3;$$
$$H_{12} = 342 = R_6(7, 3).$$

Proof. The first part follows directly from Corollary 2.1 with $s = 7$. If $b = (40d/9) + 1$, then $n = 3(1 + ((40d/9) + 1)^{m/2})/10$ for odd $m \geq 1$ is a solution that satisfies (8) identically when $d = H_{3k}$ for $k \in \mathbb{Z}^+$.

As before, Equation (8) reduces to

$$y^2 = x^3 + k, \tag{9}$$

where now

$$y := 5b^r d(b-1)^2(10n-3), \quad x := 10b^{m_1+r}d(b-1),$$

and

$$k := 25b^{2r}d^2(b-1)^3(9b-40d-9).$$

Notice that $k \neq 0$. We now use *MAGMA* [3] to determine all integer points of the 108 Mordell curves given by (9). In the following Table 5, we can find the integer points of the two elliptic curves that give rise to non-trivial solutions of Equation (8). The remaining 106 elliptic curves either have no integer points, or have integer points that do not produce non-trivial positive integer solutions of Equation (8), and we omit them.

Table 5. Integer solutions (x, y).

$\boldsymbol{b}$	$\boldsymbol{d}$	$\boldsymbol{r}$	$\boldsymbol{k}$	$\boldsymbol{(x,y)}$	$\boldsymbol{(m,n)}$
2	1	0	-775	$(10, \pm 15), (19, \pm 78), (\mathbf{20}, -\mathbf{85}), (20, 85),$ $(70, \pm 585), (80, 715), (80, -715),$ $(16750, \pm 2167815), (26530, \pm 4321215)$	$(3, 2)$
7	6	0	-36158400	$(360, \pm 3240), (369, \pm 3753), (684, \pm 16848),$ $(720, \pm 18360), (\mathbf{2520}, -\mathbf{126360}),$ $(2520, 126360), (2800, \pm 148040),$ $(2880, \pm 154440), (603000, \pm 468248040),$ $(955080, \pm 933382440), (14290281, \pm 54020817621)$	$(3, 12)$

Again, the integer points (x, y) in **bold** correspond to the pair (m, n) in the sixth column. The values of b and d in the first and second columns together with the corresponding values of m and n in the sixth column form the non-trivial solutions of equation (8). This concludes the proof of Corollary 2.7. $\square$

3 Conclusion

We have proved a finiteness theorem about b-repdigits that can be represented as a quadratic polynomial with rational coefficients. With the help of this result, we have proved that there is only a finite number of polygonal numbers which are b-repdigits except in some cases that we characterize. Finally, we have illustrated this result for the particular cases of triangular, heptagonal and pentagonal numbers, finding all numbers of this type in the sequence of b-repdigits for $b \in [2, 9]$.

In 2018, Kafle, Luca, and Togbé [5] also proved the finiteness of the solutions of some equations that involves *repblocks of two digits*. Such numbers have the form

$$d\left(\frac{10^{2m}-1}{99}\right),$$

for some $m \geq 1$ and $d \in \{10, 11, \ldots, 99\}$. As a consequence of their result, they found all repblocks of two digits that are triangular numbers. Now, we extend the notion of *repblocks of two digits* and consider the positive integer in base $b \geq 2$ with repeated blocks of two digits, which we call *b-repblocks of two digits*. Such a number has the form

$$d\left(\frac{b^{2m}-1}{b^2-1}\right),$$

for some $m \geq 1$ and $d \in \{b, b+1, \ldots, b^2-1\}$. As a research we propose to generalize the main result of Kafle, Luca, and Togbé [5, Theorem 1] by considering b-repblocks of two digits instead of repblocks of two digits.

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