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New Fibonacci-type pulsated sequences. Part 2

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Abstract: A new Fibonacci sequence from a pulsated type is introduced. The explicit form of its members is given.
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1 Introduction

The present work is a continuation of the previous authors' paper [17]. As pointed out in the first part of the research [17], the first extension of the Fibonacci sequence in the form of two or more sequences, was introduced in [11]. This idea has been developed in different directions (see, e.g., [2–14, 16, 18–22]). In the first part of the research, we studied three Fibonacci-type sequences of the so called pulsated sequences. In the present study, another form of three pulsated Fibonacci-type sequences will be discussed, and a property of theirs wil be proven.

2 Main results

Let a, b, c, d be fixed real numbers. Let us define the following Fibonacci sequence of pulsated type:

$$\begin{aligned}
\alpha_0 &= a,\\
\beta_0 &= b,\\
\gamma_0 &= c,\\
\beta_1 &= d,\\
\alpha_{2k+2} &= \beta_{2k+1} + \gamma_{3k},\\
\beta_{2k+2} &= \beta_{2k+1} + \beta_{3k},\\
\gamma_{2k+2} &= \beta_{2k+1} + \alpha_{3k},\\
\beta_{2k+3} &= \beta_{2k+2} + \beta_{3k+1},
\end{aligned}$$

where $k \geq 0$ is an integer. The first members of this new sequence are as given in the following Table 1.

Table 1. The first members of the pulsated Fibonacci sequence

n	α_n	β_n	γ_n
0	a	b	c
1		d	
2	$c+d$	$b+d$	$a+d$
3		$b+2d$	
4	$a+b+3d$	$2b+3d$	$b+c+3d$
5		$3b+5d$	
6	$4b+c+8d$	$5b+8d$	$a+4b+8d$
7		$8b+13d$	
8	$a+12b+21d$	$13b+21d$	$12b+c+21d$
9		$21b+34d$	
10	$33b+c+55d$	$34b+55d$	$a+33b+55d$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Theorem 1. *For every four real numbers a, b, c, d and for every integer $k \geq 0$:*

$$\begin{aligned}
\alpha_{4k+2} &= (F_{4k+1} - 1)b + c + F_{4k+2}d,\\
\beta_{4k+2} &= F_{4k+1}b + F_{4k+2}d,\\
\gamma_{4k+2} &= a + (F_{4k+1} - 1)b + F_{4k+2}d,\\
\beta_{4k+3} &= F_{4k+2}b + F_{4k+3}d,\\
\alpha_{4k+4} &= (F_{4k+3} - 1)b + c + F_{4k+4}d,\\
\beta_{4k+4} &= F_{4k+3}b + F_{4k+4}d,\\
\gamma_{4k+4} &= a + (F_{4k+3} - 1)b + F_{4k+4}d,\\
\beta_{4k+5} &= F_{4k+4}b + F_{4k+5}d.
\end{aligned}$$

Proof. For $k = 0, 1$ the assertion is valid (see the above Table 1). Let us assume that it is valid for some k. Then:

$$\begin{aligned}
\alpha_{4(k+1)+2} &= \alpha_{4k+6} \\
&= \beta_{4k+5} + \gamma_{4k+4}, \\
&= F_{4k+4}b + F_{4k+5}d + a + (F_{4k+3} - 1)b + F_{4k+4}d \\
&= a + F_{4k+4}b + (F_{4k+5} - 1)b + F_{4k+6}d \\
\beta_{4(k+1)+2} &= \beta_{4k+6} \\
&= \beta_{4k+5} + \beta_{4k+4}, \\
&= F_{4k+4}b + F_{4k+5}d + F_{4k+3}b + F_{4k+4}d \\
&= F_{4k+5}b + F_{4k+6}d \\
\gamma_{4(k+1)+2} &= \gamma_{4k+6} \\
&= \beta_{4k+5} + \alpha_{4k+4}, \\
&= F_{4k+4}b + F_{4k+5}d + (F_{4k+3} - 1)b + c + F_{4k+4}d \\
&= (F_{4k+5} - 1)b + c + F_{4k+6}d \\
\beta_{4(k+1)+3} &= \beta_{4k+7} \\
&= \beta_{4k+6} + \beta_{4k+5}, \\
&= F_{4k+5}b + F_{4k+6}d + F_{4k+4}b + F_{4k+5}d \\
&= F_{4k+6}b + F_{4k+7}d.
\end{aligned}$$

□

In [1], the arithmetic function ψ is defined and in [15] it is applied over the members of the Fibonacci numbers, proving that they have a basis with 24 elements about the ψ-function, as follows (see Table 2).

Table 2. The basis of 24 elements of the Fibonacci and the newly proposed sequences

n	0	1	2	3	4	5	6	7	8	9	10	11
$\psi(n)$	0	1	1	2	3	5	8	4	3	7	1	8
n	12	13	14	15	16	17	18	19	20	21	22	23
$\psi(n)$	9	8	8	7	6	4	1	5	6	2	8	1
n	24	25	26	27	28	29	30	31	32	33	34	...
$\psi(n)$	9	1	1	2	3	5	8	4	3	7	1	...

Let us define for every two natural numbers $k \geq 0$ and i $(0 \leq i \leq 3)$:

$$f_{4k+i} = \psi(F_{4k+i}).$$

Then the above table can obtain the form for the sequence $\{f_{4k+i}\}$ (see Table 3).

Table 3. Alternative presentation

$i\backslash k$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	...
0	0	3	3	9	6	6	9	...
1	1	5	7	8	4	2	1	...
2	1	8	1	8	1	8	1	...
3	2	4	8	7	5	1	2	...

Hence, the newly proposed type of Fibonacci-type pulsated sequence has the same basis with 24 elements. Therefore, the above Theorem can be modified to the following form.

Theorem 2. *For every four real numbers a, b, c, d and for every integer $k \geq 0$:*

$$
\begin{aligned}
\psi(\alpha_{4k+2}) &= (f_{4k+1} - 1)b + c + f_{4k+2}d,\\
\psi(\beta_{4k+2}) &= f_{4k+1}b + f_{4k+2}d,\\
\psi(\gamma_{4k+2}) &= a + (f_{4k+1} - 1)b + f_{4k+2}d,\\
\psi(\beta_{4k+3}) &= f_{4k+2}b + f_{4k+3}d,\\
\psi(\alpha_{4k+4}) &= (f_{4k+3} - 1)b + c + f_{4k+4}d,\\
\psi(\beta_{4k+4}) &= f_{4k+3}b + f_{4k+4}d,\\
\psi(\gamma_{4k+4}) &= a + (f_{4k+3} - 1)b + f_{4k+4}d,\\
\psi(\beta_{4k+5}) &= f_{4k+4}b + f_{4k+5}d.
\end{aligned}
$$

3 Conclusion

The Fibonacci-type pulsated sequences discussed in the paper have a new form, different from the previously defined ones. In a future leg of the present research, other Fibonacci-type sequences that have a changing form will be defined and studied.

References

[1] Atanassov, K. (1985). An arithmetic function and some of its applications. *Bulletin of Number Theory and Related Topics*, 9, 18–27.

[2] Atanassov, K. (1986). On a second new generalization of the Fibonacci sequence. *The Fibonacci Quarterly*, 24(4), 362–365.

[3] Atanassov, K. (1989). On a generalization of the Fibonacci sequence in the case of three sequences. *The Fibonacci Quarterly*, 27(1), 7–10.

[4] Atanassov, K. (2010). Combined 2-Fibonacci sequences. *Notes on Number Theory and Discrete Mathematics*, 16(2), 24–28.

[5] Atanassov, K. (2013). Pulsating Fibonacci sequences. *Notes on Number Theory and Discrete Mathematics*, 19(3), 12–14.

[6] Atanassov, K. (2013). Pulsated Fibonacci sequence. Part 2. *Notes on Number Theory and Discrete Mathematics*, 19(4), 33–36.

[7] Atanassov, K. (2014). n-Pulsated Fibonacci sequence. *Notes on Number Theory and Discrete Mathematics*, 20(1), 32–35.

[8] Atanassov, K. (2017). On two new two-dimensional extensions of the Fibonacci sequence. *Notes on Number Theory and Discrete Mathematics*, 23(3), 115–122.

[9] Atanassov, K. (2018). On two new combined 3-Fibonacci sequences. *Notes on Number Theory and Discrete Mathematics*, 24(2), 90–93.

[10] Atanassov, K. (2021). A short remark on a new Fibonacci-type sequence. *Notes on Number Theory and Discrete Mathematics*, 27(2), 168–171.

[11] Atanassov, K., Atanassova, L. & Sasselov, D. (1985). A new perspective to the generalization of the Fibonacci sequence. *The Fibonacci Quarterly*, 23(1), 21–28.

[12] Atanassov, K., Atanassova, V., Shannon, A., & Turner, J. (2002). *New Visual Perspectives on Fibonacci Numbers*. World Scientific, New Jersey.

[13] Atanassov, K., Deford, D. R., & Shannon, A. G. (2014). Pulsated Fibonacci recurrences. In: *Anderson, P., Ballot, C., & Webb, W. (Eds.). Proceedings of the Sixteenth International Conference on Fibonacci Numbers and Their Applications*, July 20–27 2014, Rochester, New York, 22–27.

[14] Atanassov, K., & Dimitrov, D. (2010). On ψ-function and 2-Fibonacci sequences. *Notes on Number Theory and Discrete Mathematics*, 16(1), 5–48.

[15] Atanassov, K., & Shannon, A. G. (2005). Digit sum bases for Fibonacci and related numbers. *Notes on Number Theory and Discrete Mathematics*, 11(3), 25–32.

[16] Atanassov, K., & Shannon, A. G. (2016). Combined 3-Fibonacci sequences from a new type. *Notes on Number Theory and Discrete Mathematics*, 22(3), 5–8.

[17] Atanassova, L., & Andonov, V. (2023). New Fibonacci-type pulsated sequences. *Notes on Number Theory and Discrete Mathematics*, 29(4), 789–793.

[18] Lee, J.-Z., & Lee, J.-S. (1987). Some properties of the generalization of the Fibonacci sequence. *The Fibonacci Quarterly*, 25(2), 111–117.

[19] Spickerman, W., & Creech, R. (1997). The $(2, T)$ generalized Fibonacci sequences. *The Fibonacci Quarterly*, 35(4), 358–360.

[20] Spickerman, W., Creech, R., & Joyner, R. (1993). On the structure of the set of difference systems defining $(3, F)$ generalized Fibonacci sequence. *The Fibonacci Quarterly*, 31(4), 333–337.

[21] Spickerman, W., Creech, R., & Joyner, R. (1995). On the $(3, F)$ generalizations of the Fibonacci sequence. *The Fibonacci Quarterly*, 33(1), 9–12.

[22] Spickerman, W., Joyner, R., & Creech, R. (1992). On the $(2, F)$-generalizations of the Fibonacci sequence. *The Fibonacci Quarterly*, 30(4), 310–314.