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Editorial Correction to "A note on a generalization of Riordan's combinatorial identity via a hypergeometric series approach" [Notes on Number Theory and Discrete Mathematics, 2023, Volume 29, Number 3, Pages 421–425]

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Correction

The expression which Lim [1, Equation (7)] gave as Knuth's old sum is wrong.

The correct identity is (see for example [2, Problem 4, p.71])

$$\sum_{k=0}^{n}(-1)^k\binom{n}{k}2^{-k}\binom{2k}{k}=\begin{cases}2^{-n}\binom{n}{n/2}, & \text{if } n \text{ is even;}\\ 0, & \text{if } n \text{ is odd.}\end{cases} \tag{7}$$

Lim's Equation (8) stating Riordan's result [2, p.72] is also not correct. The correct identity is

$$\sum_{k=0}^{n}(-1)^k\binom{n+1}{k+1}2^{-k}\binom{2k}{k}=\begin{cases}\dfrac{(3/2)_{n/2}}{(1)_{n/2}}, & \text{if } n \text{ is even;}\\ \dfrac{(3/2)_{(n-1)/2}}{(1)_{(n-1)/2}}, & \text{if } n \text{ is odd;}\end{cases} \tag{8}$$

where $(z)_j$ is the Pochhammer symbol defined for a positive integer j and a complex number z by $(z)_j = z(z+1)\cdots(z+j-1)$.

Identity (8) can be written more briefly as

$$\sum_{k=0}^{n}(-1)^k\binom{n+1}{k+1}2^{-k}\binom{2k}{k}=\frac{\left(\frac{3}{2}\right)_{\lfloor n/2\rfloor}}{(1)_{\lfloor n/2\rfloor}},$$

where $\lfloor z\rfloor$ is the greatest integer less than or equal to z.

Lim's main result, Equation (9) of his Theorem 2.1 is also wrong. The correct result is

$$\sum_{k=0}^{n}(-1)^k\binom{n+i}{k+i}2^{-k}\binom{2k}{k}\frac{(i+1)_k}{(2)_k}=\binom{n+i}{i}\frac{\left(\frac{1}{2}\right)_{\lceil n/2\rceil}}{(1)_{\lceil n/2\rceil}},\tag{9}$$

where $\lceil z\rceil$ is the smallest integer greater than or equal to z.

Finally, Corrolaries (10) and (11) are wrong. The correct results are

$$\sum_{k=0}^{n}(-1)^k\binom{n+2}{k+2}2^{-k}\binom{2k}{k}\frac{(3)_k}{(2)_k}=\begin{cases}\binom{n+2}{2}\frac{\left(\frac{1}{2}\right)_{n/2}}{(1)_{n/2}}, & \text{if } n \text{ is even},\\ \binom{n+2}{2}\frac{\left(\frac{1}{2}\right)_{(n+1)/2}}{(1)_{(n+1)/2}}, & \text{if } n \text{ is odd},\end{cases}\tag{10}$$

and

$$\sum_{k=0}^{n}(-1)^k\binom{n+3}{k+3}2^{-k}\binom{2k}{k}\frac{(4)_k}{(2)_k}=\begin{cases}\binom{n+3}{3}\frac{\left(\frac{1}{2}\right)_{n/2}}{(1)_{n/2}}, & \text{if } n \text{ is even},\\ \binom{n+3}{3}\frac{\left(\frac{1}{2}\right)_{(n+1)/2}}{(1)_{(n+1)/2}}, & \text{if } n \text{ is odd}.\end{cases}\tag{11}$$

References

[1] Lim, D. (2023). A note on a generalization of Riordan's combinatorial identity via a hypergeometric series approach. *Notes on Number Theory and Discrete Mathematics*, 29(3), 421–425.

[2] Riordan, J. (1971). *Combinatorial Identities*. John Wiley & Sons, Inc., New York.

Note from the Editorial Office

The presented correction notice to [1] *was submitted to the NNTDM Editorial Office on 26 March 2024 by the Editorial Board member Prof. Kunle Adegoke. Numerous attempts were made in April 2024 to reach the author Dongkyu Lim and respectfully give him the courtesy and opportunity to prepare by himself a corrigendum to his paper on the basis of the comments provided by Prof. Adegoke.*

As no response or acknowledgement of receipt of these emails has been received by the end of April 2024, the Editorial Office publishes the present Editorial Correction as is.

We believe that it is in the best interest of the readers of the paper [1]*, of the Journal "Notes on Number Theory and Discrete Mathematics", and of the paper's author Dongkyu Lim that the correction notice regarding the errors found in the paper are announced and rectified in the most timely and efficient manner.*