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Corrigendum to "On upper Hermite–Hadamard inequalities for geometric-convex and log-convex functions" [Notes on Number Theory and Discrete Mathematics, 2014, Vol. 20, No. 5, 25–30]

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In paper [1], Lemma 2.3 states that if $a, b, p, q > 0$ are real numbers and $\frac{q}{p} \geq \frac{b}{a} \geq 1$, then

$$L(pa, qb) \leq L(p, q).A(a, b), \tag{1}$$

where $L(x, y)$ and $A(x, y)$ denote, as usual, the logarithmic, respectively arithmetic, means of $x, y > 0$. In fact, inequality (1) holds true with reversed order of inequality (i.e., "$\geq$" instead of "$\leq$"). By letting $u = \frac{q}{p}$ and $v = \frac{b}{a}$, it is immediate that it is sufficient to prove that

$$\frac{uv - 1}{\ln(uv)} > \frac{v + 1}{2}.\frac{u - 1}{\ln(u)} \qquad (u > v > 1). \tag{2}$$

To prove (2), let

$$k(u) = 2(uv - 1)\ln u - (v + 1)(u - 1)\ln(uv),$$

where $v > 1$ is fixed. After simple computations, we get

$$k'(u) = (v - 1).\left[\ln u + 1 + \frac{1}{u}\right] - (v + 1)\ln v.$$

As

$$\left(\ln u + 1 + \frac{1}{u}\right)' = \frac{u - 1}{u^2} > 0$$

for $u > 1$, it follows that

$$k'(u) > (v-1)(\ln v + 1 + \frac{1}{v}) - (v+1)\ln v = -\ln(v^2) + \frac{v^2-1}{v} > 0$$

on the basis of inequality $L(v^2, 1) > \sqrt{v^2.1}$ (i.e., $L > G$, where G is the geometric mean). Thus we get $k(u) > 0$ for $u > v$, and this finishes the proof of (2).

Remark. Therefore, the last inequality in Theorem 2.5 of [1] holds true with reversed order, i.e.,

$$\frac{1}{A(a,b)}.L(af(a), bf(b)) \geq L(f(a), f(b)), \tag{3}$$

by assuming the conditions of Theorem 2.5.

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References

[1] Sándor, J. (2014). On upper Hermite–Hadamard inequalities for geometric-convex and log-convex functions. *Notes on Number Theory and Discrete Mathematics*, 20(5), 25–30.