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Corrigendum to "The Oresme sequence: The generalization of its matrix form and its hybridization process" [Notes on Number Theory and Discrete Mathematics, Vol. 27, 2021, No. 1, 101–111]

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The present Corrigendum contains a list of corrections applicable to the authors' paper [1].

1. In the **Introducion** part, where was written "$O_n = -n2^n$, with n being a positive integer" replace for "$O_n = n \cdot 2^{-n}$, for an integer $n \geq 0$".

2. For **Theorems 2.1, 2.2 and 2.4**, where was written:

$$\text{For } O = \begin{bmatrix} 1 & -\frac{1}{4} \\ 1 & 0 \end{bmatrix} \text{ we have that: } O^n = \begin{bmatrix} 2O_{n+1} & -\frac{1}{2}O_n \\ 2O_n & -\frac{1}{2}O_{n-1} \end{bmatrix}^n, n \geqslant 1;$$

For $O = \begin{bmatrix} 0 & 1 \\ -\frac{1}{4} & 1 \end{bmatrix}$, we have that: $O^n = \begin{bmatrix} -\frac{1}{2}O_{n-1} & 2O_n \\ -\frac{1}{2}O_n & 2O_{n+1} \end{bmatrix}^n$, $n \geqslant 1$.

For $\sigma = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix}$, we have to: $\sigma^n = \begin{bmatrix} 2O_{-n+1} & -\frac{1}{2}O_{-n} \\ 2O_{-n} & -\frac{1}{2}O_{-n-1} \end{bmatrix}^n$, for $n > 0$.

replace with:

For $O = \begin{bmatrix} 1 & -\frac{1}{4} \\ 1 & 0 \end{bmatrix}$, we have that: $O^n = \begin{bmatrix} 2O_{n+1} & -\frac{1}{2}O_n \\ 2O_n & -\frac{1}{2}O_{n-1} \end{bmatrix}$, $n \geqslant 1$;

For $O = \begin{bmatrix} 0 & 1 \\ -\frac{1}{4} & 1 \end{bmatrix}$, we have that: $O^n = \begin{bmatrix} -\frac{1}{2}O_{n-1} & 2O_n \\ -\frac{1}{2}O_n & 2O_{n+1} \end{bmatrix}$, $n \geqslant 1$;

For $\sigma = \begin{bmatrix} 0 & 1 \\ -4 & 4 \end{bmatrix}$, we have to: $\sigma^n = \begin{bmatrix} 2O_{-n+1} & -\frac{1}{2}O_{-n} \\ 2O_{-n} & -\frac{1}{2}O_{-n-1} \end{bmatrix}$, for $n > 0$.

3. In **Definition 2.3**, where was written "$n < 0$" replace for "$n > 0$".

4. In the **Properties inherent to matrices** section, it is verified that the presented properties are valid for $n = 1$ but it is not possible to obtain the same result when modifying n. Thus, in this errata, we present properties that are valid for every n assigned:

Property 2.6. For any integer $n, r, 1 \leqslant n < r$, we have:

$$O_{n+r} = 2O_nO_{r+1} - \frac{1}{2}O_{n-1}O_r.$$

Proof. According to Theorem 2.1 and some properties for square matrix exponents, we have:

$$\begin{aligned} O^{n+r} &= O^nO^r, \\ \begin{bmatrix} 2O_{n+r+1} & -\frac{1}{2}O_{n+r} \\ 2O_{n+r} & -\frac{1}{2}O_{n+r-1} \end{bmatrix} &= \begin{bmatrix} 2O_{n+1} & -\frac{1}{2}O_n \\ 2O_n & -\frac{1}{2}O_{n-1} \end{bmatrix} \begin{bmatrix} 2O_{r+1} & -\frac{1}{2}O_r \\ 2O_r & -\frac{1}{2}O_{r-1} \end{bmatrix}. \end{aligned}$$

Considering the element a_{21} we have:

$$\begin{aligned} 2O_{n+r} &= 2O_n2O_{r+1} - \frac{1}{2}O_{n-1}2O_r, \\ O_{n+r} &= 2O_nO_{r+1} - \frac{1}{2}O_{n-1}O_r. \end{aligned}$$

□

Note 2.7. On Property 2.6, if $n = 1$, we have:

$$\begin{aligned} O_{n+r} &= 2O_nO_{r+1} - \frac{1}{2}O_{n-1}O_r, \\ O_{r+1} &= 2O_1O_{r+1} - \frac{1}{2}O_0O_r, \\ O_{r+1} &= 2\frac{1}{2}O_{r+1} - \frac{1}{2}0O_r, \\ O_{r+1} &= O_{r+1}. \end{aligned}$$

Property 2.8. For any integer $m, r, 0 < m < r$, we have to:

$$O_{n+r+1} = 2O_{n+1}O_{r+1} - \frac{1}{2}O_nO_r.$$

Proof. According to Property 2.6 and the element a_{11}, we have:

$$\begin{aligned} 2O_{n+r+1} &= 2O_{n+1}2O_{r+1} - \frac{1}{2}O_n2O_r, \\ O_{n+r+1} &= 2O_{n+1}O_{r+1} - \frac{1}{2}O_nO_r. \end{aligned}$$ □

Note 2.9. On Property 2.8, if $n = 1$, we have to:

$$\begin{aligned} O_{n+r+1} &= 2O_{n+1}O_{r+1} - \frac{1}{2}O_nO_r, \\ O_{r+2} &= 2O_2O_{r+1} - \frac{1}{2}O_2O_r, \\ O_{r+2} &= 2\frac{2}{4}O_{r+1} - \frac{1}{2}.\frac{2}{4}O_r, \\ O_{r+2} &= O_{r+1} - \frac{1}{4}O_r. \end{aligned}$$

Property 2.10. For any integer $n, r, 0 < n < r$, we have:

$$O_{-n-r} = 2O_{-n}O_{-r+1} - \frac{1}{2}O_{-n-1}O_{-r}.$$

Proof. According to Theorem 2.4 and some properties for square matrix exponents, we have:

$$\begin{aligned} \sigma^{n+r} &= \sigma^n\sigma^r, \\ \begin{bmatrix} 2O_{-n-r+1} & -\frac{1}{2}O_{-n-r} \\ 2O_{-n-r} & -\frac{1}{2}O_{-n-r-1} \end{bmatrix} &= \begin{bmatrix} 2O_{-n+1} & -\frac{1}{2}O_{-n} \\ 2O_{-n} & -\frac{1}{2}O_{-n-1} \end{bmatrix} \begin{bmatrix} 2O_{-r+1} & -\frac{1}{2}O_{-r} \\ 2O_{-r} & -\frac{1}{2}O_{-r-1} \end{bmatrix}. \end{aligned}$$

Considering the left and right elements, we have:

$$\begin{aligned} 2O_{-n-r} &= 2O_{-n}2O_{-r+1} - \frac{1}{2}O_{-n-1}2O_{-r}, \\ O_{-n-r} &= 2O_{-n}O_{-r+1} - \frac{1}{2}O_{-n-1}O_{-r}. \end{aligned}$$ □

Note 2.11. In Property 2.11, if $n = 1$, we have:

$$\begin{aligned} O_{-n-r} &= 2O_{-n}O_{-r+1} - \frac{1}{2}O_{-n-1}O_{-r}, \\ O_{-1-r} &= 2O_{-1}O_{-r+1} - \frac{1}{2}O_{-2}O_{-r}, \\ O_{-r-1} &= -4O_{-r+1} + 4O_{-r}, \\ O_{-r-1} &= 4O_{-r} - 4O_{-r+1}. \end{aligned}$$

Property 2.12. For any integer $n, r, 0 < n < r$, we have:

$$O_{-n-r+1} = 2O_{-n+1}O_{-r+1} - \frac{1}{2}O_{-n}O_{-r}.$$

Proof. According to Property 2.10 and and the element a_{11}, we have:

$$\begin{aligned} 2O_{-n-r+1} &= 2O_{-n+1}2O_{-r+1} - \frac{1}{2}O_{-n}2O_{-r}, \\ O_{-n-r+1} &= 2O_{-n+1}O_{-r+1} - \frac{1}{2}O_{-n}O_{-r}. \end{aligned}$$

□

Note 2.13. In Property 2.12, if $n = 1$, we have:

$$\begin{aligned} O_{-n-r+1} &= 2O_{-n+1}O_{-r+1} - \frac{1}{2}O_{-n}O_{-r}, \\ O_{-r} &= 2O_0O_{-r+1} - \frac{1}{2}O_{-1}O_{-r}, \\ O_{-r} &= 2(0)O_{-r+1} - \frac{1}{2}(-2)O_{-r}, \\ O_{-r} &= O_{-r}. \end{aligned}$$

5. And yet, it was possible to identify some references that are not mentioned throughout the text, they are [2–7].

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