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Book review: "Arithmetic Functions"

Anthony Greville Shannon

Warrane College, The University of New South Wales
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Arithmetic Functions
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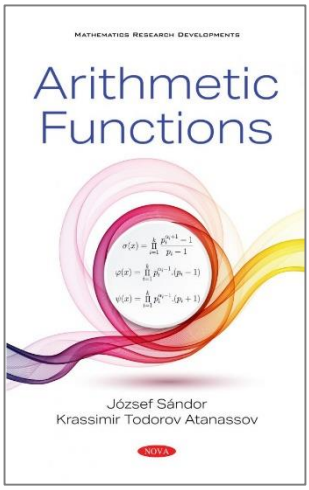

This is an attractively produced book with easy to read typography. The cover is simple and features the divisor function $\sigma(x)$, Euler's totient function $\varphi(x)$, and Dedekind's function $\psi(x)$, from which grow the four substantial chapters of the book:

1. On standard arithmetic functions $\varphi(x)$, $\psi(x)$ and $\sigma(x)$
2. Perfect and related numbers
3. On modifications and extensions of the arithmetic functions φ, ψ and σ
4. Arithmetic functions of other types.

As well as a table of Contents, there are also a Glossary of symbols, About the authors and Index, so that the book is quite self-contained which, as well as the topics and their treatment, would make it quite suitable for senior undergraduates in a mathematics or computing degree (for the topics lend themselves to computational investigations). As we shall see in this review, there is also much to attract graduate students and their advisors, particularly in the 155 references which are cited. Each section in the book brings in one or more of the three basic arithmetic functions as well as their extensions and generalizations, and classical number theoretic results such as the Cauchy–Bunyakowski inequality.

This is particularly true of Chapter 2 which builds on variations of Perfect Numbers, as well as Harmonic, Hyperperfect and Balanced Numbers. In this, it builds on some of the pioneering pure mathematical results of Graeme Cohen and the computational skills of his former research student, Ron Sorli, from the University of Technology Sydney. The results in the book are generally known, but they are collated within a critical framework, which suggests problems for further research.

Other chapters follow a similar pattern: old results connected so that new conclusions emerge, not so much from chronological connections but from deeper questions in the original problems. While Chapter 1 sets the scene for development of subsequent ideas, Chapter 3 displays many tables, particularly those which deal with Restrictive and Extension factors of natural numbers, defined respectively from the canonical factorization for n

$$n = \prod_{i=1}^{k} p_i^{\alpha_i}, \text{ by } RF(n) = \prod_{i=1}^{k} p_i^{\alpha_i - 1} \text{ and } EF(n) = \prod_{i=1}^{k} p_i^{\alpha_i + 1}.$$

Chapter 4 brings the theme of arithmetical functions neatly together with various conjectures connected with such names as Euler, Kashiihara, Klamkin, Luca, Mascheroni and Sándor. The last named has been working with Atanassov since the mid-1980s which was about the time that this reviewer also started to work with him. Lest this be seen as a conflict of interest, one thing about the book which annoyed me was the occasional mismatch between names in the Index and their actual page numbers in the book! That said, I do not hesitate to say that this book is a valuable and worthy contribution to the literature of 21st Century number theory from these Romanian and Bulgarian authors.