Notes on Number Theory and Discrete Mathematics
Print ISSN 1310–5132, Online ISSN 2367–8275
Vol. 27, 2021, No. 2, 168–171
DOI: 10.7546/nntdm.2021.27.2.168-171

A short remark on a new Fibonacci-type sequence

Krassimir T. Atanassov[1,2]

[1] Department of Bioinformatics and Mathematical Modelling
IBPhBME – Bulgarian Academy of Sciences,
Acad. G. Bonchev Str. Bl. 105, Sofia-1113, Bulgaria
e-mail: krat@bas.bg

[2] Intelligent Systems Laboratory
Prof. Dr Asen Zlatarov University, Burgas-8010, Bulgaria

Received: 1 January 2021 **Accepted:** 4 May 2021

Abstract: A new Fibonacci-type sequence is constructed and for it is proved that it has a basis with 24 elements.

Keywords: Arithmetic function, Fibonacci sequence.
2020 Mathematics Subject Classification: 11B39.

1 Introduction

In this short remark, we describe a sequence generated by two arbitrary one-digit natural numbers and the arithmetic function ψ, defined and studied in [1, 2].

Everywhere we use the natural number n of the following form

$$n = \sum_{i=1}^{k} a_i.10^{k-i} \equiv \overline{a_1 a_2 ... a_k},$$

where a_i is a natural number and $0 \leq a_i \leq 9$ $(1 \leq i \leq k)$.

Following [1, 2], we define a function denoted by φ by:

$$\varphi(n) = \begin{cases} 0, & \text{if } n = 0, \\ \sum\limits_{i=1}^{k} a_i, & \text{if } n > 0. \end{cases}$$

Now, we define a sequence of functions $\varphi_0, \varphi_1, \varphi_2, \dots$, where for each natural number l:

$$\varphi_0(n) = n,$$
$$\varphi_{l+1} = \varphi(\varphi_l(n)).$$

Obviously, for every $l \in \mathbb{N}$, $\varphi_l : \mathbb{N} \to \mathbb{N}$. Since for $k > 1$

$$\varphi(n) = \sum_{i=1}^{k} a_i < \sum_{i=1}^{k} a_i.10^{k-i} = n,$$

then for every $n \in \mathbb{N}$ there exists $l \in \mathbb{N}$ such that

$$\varphi_l(n) = \varphi_{l+1}(n) \in \Delta \equiv \{0, 1, 2, ..., 9\}.$$

Following [1,2], let the function ψ be defined by

$$\psi(n) = \varphi_l(n),$$

where

$$\varphi_{l+1}(n) = \varphi_l(n).$$

Hence, $\psi : \mathbb{N} \to \Delta$.

The following equalities are proved in [1,2] for every two natural numbers m and n:

$$\psi(0) = 0$$
$$\psi(m + n) = \psi(\psi(m) + \psi(n)),$$
$$\psi(n + 9) = \psi(n).$$

2 Main result

Now, having in mind that

$$\psi(10m + n) = \psi(m + n)$$

(see [1,2]), we obtain sequentially for two arbitrary natural numbers $a, b \in \{0, 1, \dots, 9\}$:

1	a
2	b
3	$\psi(\overline{ba}) = \psi(10b + a) = \psi(a + b)$
4	$\psi(b + \psi(a + b)) = \psi(a + 2b)$
5	$\psi(\psi(a + b) + \psi(a + 2b)) = \psi(2a + 3b)$
6	$\psi(\psi(a + 2b) + \psi(2a + 3b)) = \psi(3a + 5b)$
7	$\psi(5a + 8b)$
8	$\psi(8a + 13b) = \psi(8a + 4b)$
9	$\psi(13a + 12b) = \psi(4a + 3b)$
10	$\psi(3a + 7b)$
11	$\psi(7a + b)$
12	$\psi(a + 8b)$
13	$\psi(8a + 9b) = \psi(8a)$

Cont'd

14	$\psi(9a+8b)=\psi(8b)$
15	$\psi(8a+8b)$
16	$\psi(8a+7b)$
17	$\psi(7a+6b)$
18	$\psi(6a+4b)$
19	$\psi(4a+b)$
20	$\psi(a+5b)$
21	$\psi(5a+6b)$
22	$\psi(6a+2b)$
23	$\psi(2a+8b)$
24	$\psi(8a+b)$
25	$\psi(a+9b)=\psi(a)=a$
26	$\psi(9a+b)=\psi(b)=b.$

Therefore, this sequence has the form

$$\alpha_0 = a,$$
$$\alpha_1 = b,$$
$$\alpha_{n+2} = \psi(10\alpha_{n+1}+\alpha_n), \text{ for } n \geq 0.$$

Let the sequence of natural numbers $a_1, a_2, \ldots$ be given and let

$$c_i = \psi(a_i)\ (i = 1, 2, \ldots).$$

Hence, following [1, 2], we deduce the sequence $c_1, c_2, \ldots$ from the former sequence. If k and l exist such that $l \geq 0$,

$$c_{i+l} = c_{k+i+l} = c_{2k+i+l} = \cdots$$

for $1 \leq i \leq k$, then we will say that

$$[c_{l+1}, c_{l+2}, \ldots, c_{l+k}]$$

is the *base of the sequence* $a_1, a_2, \ldots$ of length k and with respect to function ψ.

Therefore, the following assertion is valid.

Theorem. *The sequence*

$$a, b, \psi(10b+a), \psi(10\psi(a+b)+b), \ldots$$

has a basis of length 24.

Analogously, we can construct the other possible sequence

$$a, b, \psi(10a+b), \psi(10\psi(a+b)+a), \ldots$$

with $(n+2)$-nd term

$$\alpha_{n+2} = \psi(\alpha_{n+1}+10\alpha_n), \text{ for } n \geq 0,$$

where $\alpha_0 = a, \alpha_1 = b$. For it, the Theorem will be valid, again.

3 Conclusion

A new type of Fibonacci like sequences has been introduced. In future, the standard Fibonacci-form of these sequences will be extended to a Tribonacci, and more generally, k-bonacci-forms.

References

[1] Atanassov, K. (1985). An arithmetical function and some of its applications. *Bulletin of Number Theory and Related Topics*, IX(1), 18–27.

[2] Atanassov, K. (2015). A digital arithmetical function and some of its applications. *Proceedings of the Jangjeon Mathematical Society*, 18(4), 511–528.