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Equitable coloring on subdivision vertex join of cycle C_m with path P_n

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Abstract: Graph coloring is one of the research areas that shaped the graph theory as we know it today. An equitable coloring of a graph G is a proper coloring of the vertices of G such that color classes differ in size by at most one. The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting a new vertex into every edge of G. Let G_1 and G_2 be two graphs with vertex sets $V(G_1)$ and $V(G_2)$, respectively. The subdivision-vertex join of two vertex disjoint graphs G_1 and G_2 is the graph obtained from $S(G_1)$ and G_2 by joining each vertex of $V(G_1)$ with every vertex of $V(G_2)$. In this paper, we find the equitable chromatic number of subdivision vertex join of cycle graph with path graph.

Keywords: Equitable coloring, Subdivision graph, Subdivision vertex join.
2010 Mathematics Subject Classification: 05C15.

1 Introduction

All graphs considered in this paper are connected, finite and simple, i.e., undirected, loop less and without multiple edges, unless otherwise stated. Let $G = (V, E)$ be a graph. A vertex coloring (or simply coloring) of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color. In a vertex coloring of G, the set of vertices with the same color is called a color class. A graph G is said to be equitably k-colorable [6] if the vertex set $V(G)$ can be partitioned into k independent sets $V_1, V_2, \ldots, V_k$ such that $||V_i| - |V_j|| \leq 1$ for every i and j. In other words an equitable coloring [8] of a graph is a proper coloring of the vertices of G such that the color classes differ in size by at most one. If $|V_i| = l$ for every $i = 1, 2, \ldots, k$, then G on $n = kl$ vertices is said to be strong equitably k-colorable. The smallest integer k for which G is equitably k-colorable is called the equitable chromatic number of G and is denoted by $\chi_=(G)$.

The coloring problem is one of the most important problems in the graph theory. As an extension of proper vertex coloring, edge coloring and total coloring, the concept and some conjectures on the equitable total coloring are developed [8]. An application of equitable coloring is found in transportation problems. Here, the vertices represent garbage collection routes and two such vertices are joined by an edge when the corresponding routes should not be run on the same day. The problem of assigning one of the six days of the work to each route becomes the problem of 6-coloring of G [6]. On practical grounds it might also be desirable to have an approximately equal number of routes run on each of the six days. So we have to color the graph in the equitable way.

The notion of equitable colorability was introduced by Meyer [6]. However, an earlier work of Hajnal and Szemeredi showed that a graph G with degree $\Delta(G)$ is equitably k-colorable if $k \geq \Delta(G) + 1$. The degree of a vertex in G is the number of vertices adjacent to it. The maximum degree over all vertices in G is denoted by $\Delta(G)$.
In 1973, Meyer [6] formulated the following conjecture
Conjecture 1 Equitable Coloring Conjecture (ECC)
For any connected graph G, other than a complete graph or an odd cycle, $\chi_=(G) \leq \Delta(G)$.
We also have a stronger conjecture
Conjecture 2 Equitable Δ-Coloring Conjecture
If G is a connected graph of degree Δ, other than a complete graph, an odd cycle or a complete bipartite graph $K_{2n+1,2n+1}$ for any $n \geq 1$, then G is equitably Δ-colorable.

Definition 1.1. The *subdivision graph* $S(G)$ [5] of the graph G is obtained from G by inserting a new vertex of degree 2 on each edge of G. For $k \geq 1$, the k-th subdivision graph $S_k(G)$ is obtained from G by inserting k new vertices of degree 2 on each edge of G.

Definition 1.2. The *subdivision-vertex join* [2] of two vertex disjoint graphs G_1 and G_2 denoted by $G_1 \dot{\vee} G_2$, is the graph obtained from $S(G_1)$ and G_2 by joining each vertex of $V(G_1)$ with every vertex of $V(G_2)$.

Let C_m and P_n denote the cycle and path graph with m and n vertices, respectively. By definition of subdivision vertex join of graphs [1, 3, 7] we subdivide each edge of the cycle graph

and join each vertex of the cycle graph with every vertex of the path graph [4, 5, 8].
Throughout this paper, $\{v_i : 1 \leq i \leq m\}$,$\{u_i : 1 \leq i \leq m\}$ and $\{s_i : 1 \leq i \leq n\}$ denote the vertices of cycle, the subdivided vertices of the cycle and the vertices of the path, respectively. The total number of vertices of the subdivision vertex join graph is $2m + n$.

Example 1.3. Subdivision vertex join of a cycle graph C_3 with a path graph P_3 is given in the following Fig. 1.

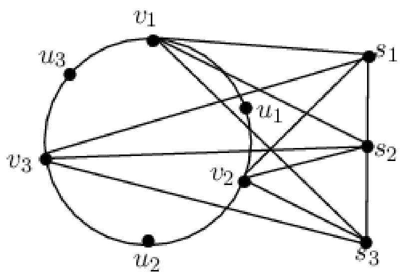

Figure 1. $C_3 \dot{\vee} P_3$

2 Main results

Theorem 2.1. *The equitable chromatic number of subdivision vertex join of a cycle graph C_m and a path graph P_n is given by the following formulas:*

1. $\chi_{=}(C_m \dot{\vee} P_n) = 3\ if\ 0 \leq |m - n| \leq 2.$
2. $\chi_{=}(C_m \dot{\vee} P_n) = 3\ if\ |m - n| \geq 3\ \ for\ \ m > n.$
3. $\chi_{=}(C_m \dot{\vee} P_n) = 4\ if\ 3 \leq |m - n| \leq \lceil \frac{m}{2} + 1 \rceil\ for\ \ m < n.$
4. $\chi_{=}(C_m \dot{\vee} P_n) = \left\lceil \dfrac{2m + n + 1}{m + 1} \right\rceil$ *otherwise for* $m < n$.

Proof. By the definition of subdivision vertex join of two graphs the edges of the cycle graph are subdivided and each vertex of the cycle graph is joined with every vertices of the path graph. The assigning of colors to the vertices of cycle, its subdivided vertices and the vertices of the path are done as follows.

1. For $0 \leq |m - n| \leq 2$.

   (a) For $|m - n| = 0$, the colors are assigned by the following subcases

   i. If m and n are even
   If $m = n = 2k$ for $k \geq 2$ we set the partition of V as follows:
   $V_1 = \{v_i : 1 \leq i \leq 2k\}$,
   $V_2 = \{u_{2i-1} : 1 \leq i \leq k\} \cup \{s_{2i-1} : 1 \leq i \leq k\}$,
   $V_3 = \{u_{2i} : 1 \leq i \leq k\} \cup \{s_{2i} : 1 \leq i \leq k\}$.

Clearly, V_1, V_2, V_3 are independent sets of $V(C_m \dot{\vee} P_n)$.
Also, $|V_1| = |V_2| = |V_3| = 2k$.
Thus, the inequality $||V_i| - |V_j|| \leq 1$ holds for every i and j.

ii. If m and n are odd
If $m = n = 2k - 1$ for $k \geq 2$ we set the partition of V as follows:
$V_1 = \{v_i : 1 \leq i \leq 2k - 1\}$
$V_2 = \{u_{2i-1} : 1 \leq i \leq k\} \cup \{s_{2i} : 1 \leq i \leq k - 1\}$
$V_3 = \{u_{2i} : 1 \leq i \leq k - 1\} \cup \{s_{2i-1} : 1 \leq i \leq k\}$

Clearly, V_1, V_2, V_3 are independent sets of $V(C_m \dot{\vee} P_n)$.
Also, $|V_1| = |V_2| = |V_3| = 2k - 1$.
Thus, the inequality $||V_i| - |V_j|| \leq 1$ hold for every i and j.

Thus (i) and (ii) show that the sets form an equitable 3-coloring of G.

(b) For $|m - n| = 1$.

The vertices $v_i : 1 \leq i \leq m$ of the cycle C_m are assigned color 1. Colors 2 and 3 are assigned, alternatively to the subdivided vertices of the cycle C_m and the path P_n (color 1 being forbidden) with the following cases.

i. m is even and n is odd
The subdivided vertices $u_i : 1 \leq i \leq m$ are assigned the colors 2 and 3, alternatively such that the colors 2 and 3 are being assigned $\frac{m}{2}$ times each. Similarly the path P_n are assigned the colors 2 and 3, alternatively with color 2 being assigned $\frac{n+1}{2}$ times and color 3 being assigned times $\frac{n-1}{2}$. Hence, the color classes are as follows:
$V_1 = \{v_1, v_2, \ldots, v_m\}$
$V_2 = \{u_1, u_3, \ldots, u_{m-1}\} \cup \{s_1, s_3, \ldots, s_n\}$
$V_3 = \{u_2, u_4, \ldots, u_m\} \cup \{s_2, s_4, \ldots, s_{n-1}\}$

$|V_1| = |V_2| = m|,\ V_3| = m - 1$, for $m > n$,
$|V_1| = |V_3| = m,\ |V_2| = m + 1$, for $m < n$.
Thus, the inequality $||V_i| - |V_j|| \leq 1$ hold for every i and j.

ii. m is odd and n is even
The subdivided vertices $u_i : 1 \leq i \leq m$ are assigned the colors 2 and 3, alternatively with color 2 being assigned $\frac{m+1}{2}$ times and color 3 being assigned $\frac{m-1}{2}$ times. The vertices of the path P_n are assigned the colors 2 and 3, alternatively with $\frac{n}{2}$ times each. Hence the color classes are as follows:
$V_1 = \{v_1, v_2, \ldots, v_m\}$
$V_2 = \{u_1, u_3, \ldots, u_m\} \cup \{s_1, s_3, \ldots, s_{n-1}\}$
$V_3 = \{u_2, u_4, \ldots, u_{m-1}\} \cup \{s_2, s_4, \ldots, s_n\}$

$|V_1| = |V_2| = m,\ |V_3| = m - 1$, for $m > n$,

$|V_1| = |V_3| = m,\ |V_2| = m + 1$, for $m < n$.
Thus, the inequality $||V_i| - |V_j|| \leq 1$ hold for every i and j.
Thus (i) and (ii) show that the sets form an equitable 3-coloring of G.

(c) For $|m - n| = 2$, the coloring is done by the following subcases

i. m and n are even
The subdivided vertices $u_i : 1 \leq i \leq m$ are assigned the colors 2 and 3, alternatively such that the colors 2 and 3 are being assigned $\frac{m}{2}$ times each. The vertices of the path P_n are assigned the colors 2 and 3, alternatively with $\frac{n}{2}$ times each. Hence the color classes are as follows:
$V_1 = \{v_1, v_2, \ldots, v_m\}$
$V_2 = \{u_1, u_3, \ldots, u_{m-1}\} \cup \{s_1, s_3, \ldots, s_{n-1}\}$
$V_3 = \{u_2, u_4, \ldots, u_m\} \cup \{s_2, s_4, \ldots, s_n\}$

$|V_1| = m, |V_2| = |V_3| = m - 1$, for $m > n$,
$|V_1| = m, |V_2| = |V_3| = m + 1$, for $m < n$.
Thus, the inequality $||V_i| - |V_j|| \leq 1$ hold for every i and j.

ii. m and n are odd
The subdivided vertices $u_i : 1 \leq i \leq m$ are assigned the colors 2 and 3, alternatively with color 2 being assigned $\frac{m+1}{2}$ times and color 3 being assigned $\frac{m-1}{2}$ times. The vertices of the path P_n are assigned the colors 2 and 3, alternatively with color 2 being assigned $\frac{n-1}{2}$ times and color 3 being assigned $\frac{n+1}{2}$ times. Hence the color classes are as follows:
$V_1 = \{v_1, v_2, \ldots, v_m\}$
$V_2 = \{u_1, u_3, \ldots, u_m\} \cup \{s_2, s_4, \ldots, s_{n-1}\}$
$V_3 = \{u_2, u_4, \ldots, u_{m-1}\} \cup \{s_1, s_3, \ldots, s_n\}$

$|V_1| = m, |V_2| = |V_3| = m - 1$, for $m > n$,
$|V_1| = m, |V_3| = |V_2| = m + 1$, for $m < n$.
Thus, the inequality $||V_i| - |V_j|| \leq 1$ hold for every i and j.

Thus, (i) and (ii) shows that the set form an equitable 3-coloring of G.
Hence $\chi_{=}(C_m \dot{\vee} P_n) = 3$ if $0 \leq |m - n| \leq 2$.

2. For $|m - n| \geq 3, m > n$

The assigning of colors equitably are done by one of the following cases

Case 1: $(2m + n) \mod 4 = 0$.

The vertices $v_i : 1 \leq i \leq \frac{2m+n}{4}$ of the cycle C_m are all assigned color 1 and the remaining vertices $v_i : \frac{2m+n}{4} + 1 \leq i \leq m$ are assigned color 2. Thus color 2 is assigned $\frac{2m-n}{4}$ times. Now we assign the colors to the subdivided vertices $u_i : 1 \leq i \leq m$. Color 2 is assigned to the subdivided vertices $u_i : 1 \leq i \leq \frac{n}{2}$ and the remaining subdivided vertices $u_i : \frac{n}{2} + 1 \leq i \leq m$ are assigned the colors 3 and 4, alternatively. Color 3 is assigned $\lceil \frac{2m-n}{4} \rceil$ times and color 4 is assigned $\lceil \frac{2m-n}{4} \rceil$ times.

The path P_n is colored with two colors 3 and 4, alternatively (colors 1 and 2 being forbidden) such that the vertices $\{s_1, s_2, \ldots, s_{n-1}, s_n\}$ are assigned the colors $\{3, 4, \ldots 3, 4\}$, respectively. Colors 3 and 4 are assigned $\frac{n}{2}$ times each.

Thus, color 1 is assigned $\frac{2m+n}{4}$ times, colors 2, 3 and 4 assigned $\frac{2m-n}{4} + \frac{n}{2}$ times each. All the four colors are assigned $\frac{2m+n}{4}$ times satisfying equitable coloring.

Case 2: $(2m + n) \mod 4 = 1.$

The vertices $v_i : 1 \leq i \leq \lfloor \frac{2m+n}{4} \rfloor$ of the cycle C_m are all assigned color 1 and the remaining vertices $v_i : \left(\lfloor \frac{2m+n}{4} \rfloor + 1\right) \leq i \leq m$ are assigned color 2. Thus color 2 is assigned $\lceil \frac{2m-n}{4} \rceil$ times.

Now, we assign the colors to the subdivided vertices $u_i : 1 \leq i \leq m.$ Color 2 is assigned to the subdivided vertices $u_i : 1 \leq i \leq \lceil \frac{n}{2} \rceil$ and the remaining subdivided vertices $u_i : \left(\lceil \frac{n}{2} \rceil + 1\right) \leq i \leq m$ are assigned the colors 3 and 4, alternatively. Color 3 is assigned $\lceil \frac{2m-n}{4} \rceil$ times and color 4 is assigned $\lceil \frac{2m-n}{4} \rceil$ times.

The path P_n is colored with two colors 3 and 4, alternatively (colors 1 and 2 being forbidden) such that the vertices $\{s_1, s_3, \ldots, s_n\}$ are assigned the colors $\{4, 3, \ldots, 3, 4\}$, respectively. Color 3 is assigned $\lfloor \frac{n}{2} \rfloor$ times and color 4 is assigned $\lceil \frac{n}{2} \rceil$ times.

Thus, color 1 is assigned $\lfloor \frac{2m+n}{4} \rfloor$ times, color 2 is assigned $\lceil \frac{2m+n}{4} \rceil$, colors 3 and 4 are assigned $\lfloor \frac{2m+n}{4} \rfloor$ each. The four colors are assigned either $\lfloor \frac{2m+n}{4} \rfloor$ or $\lceil \frac{2m+n}{4} \rceil$ satisfying equitable coloring.

Case 3: $(2m + n) \mod 4 = 2.$

The vertices $v_i : 1 \leq i \leq \lceil \frac{2m+n}{4} \rceil$ of the cycle C_m are all assigned color 1 and the remaining vertices $v_i : \lceil \frac{2m+n}{4} \rceil + 1 \leq i \leq m$ are assigned color 2. Thus color 2 is assigned $\lfloor \frac{2m-n}{4} \rfloor$ times.

Now, we assign the colors to the subdivided vertices $u_i : 1 \leq i \leq m.$ Color 2 is assigned to the subdivided vertices $u_i : 1 \leq i \leq \frac{n}{2}$ and the remaining subdivided vertices $u_i : \frac{n}{2} + 1 \leq i \leq m$ are assigned the colors 3 and 4,alternatively. Color 3 is assigned $\lceil \frac{2m-n}{4} \rceil$ times and color 4 is assigned $\lfloor \frac{2m-n}{4} \rfloor$ times.

The path P_n is colored with two colors 3 and 4, alternatively (colors 1 and 2 being forbidden) such that the vertices $\{s_1, s_3, \ldots, s_n\}$ are assigned the colors $\{4, 3, \ldots, 3, 4\}$, respectively. Color 3 is assigned $\frac{n}{2}$ times and color 4 is assigned $\frac{n}{2}$ times.

Thus, color 1 is assigned $\lceil \frac{2m+n}{4} \rceil$ times, color 2 is assigned $\lfloor \frac{2m+n}{4} \rfloor$ times, color 3 is assigned $\lceil \frac{2m+n}{4} \rceil$ times and color 4 is assigned $\lfloor \frac{2m+n}{4} \rfloor$ times. The four colors are assigned either $\lfloor \frac{2m+n}{4} \rfloor$ or $\lceil \frac{2m+n}{4} \rceil$ satisfying equitable coloring.

Case 4: $(2m + n) \mod 4 = 3.$

The vertices $v_i : 1 \leq i \leq \lceil \frac{2m+n}{4} \rceil$ of the cycle C_m are all assigned color 1 and the remaining vertices $v_i : \lceil \frac{2m+n}{4} \rceil + 1 \leq i \leq m$ are assigned color 2. Thus color 2 is assigned $\lfloor \frac{2m-n}{4} \rfloor$ times.

Now, we assign the colors to the subdivided vertices $u_i : 1 \le i \le m$. Color 2 is assigned to the subdivided vertices $u_i : 1 \le i \le \lceil \frac{n}{2} \rceil$ and the remaining subdivided vertices $u_i : \lceil \frac{n}{2} \rceil + 1 \le i \le m$ are assigned the colors 3 and 4, alternatively. Color 3 is assigned $\lfloor \frac{2m-n}{4} \rfloor$ times and color 4 is assigned $\lfloor \frac{2m-n}{4} \rfloor$ times.

The path P_n is colored with two colors 3 and 4, alternatively (colors 1 and 2 being forbidden) such that the vertices $\{s_1, s_3, \ldots, s_n\}$ are assigned the colors $\{3, 4, \ldots, 3, 4\}$, respectively. Color 3 is assigned $\lceil \frac{n}{2} \rceil$ times and color 4 is assigned $\lfloor \frac{n}{2} \rfloor$ times.

Thus, colors 1,2,3 are assigned $\lceil \frac{2m+n}{4} \rceil$ times each, color 4 is assigned $\lfloor \frac{2m+n}{4} \rfloor$ times. The four colors are assigned either $\lceil \frac{2m+n}{4} \rceil$ or $\lfloor \frac{2m+n}{4} \rfloor$ times satisfying equitable coloring.

Thus, all the above cases are equitably 4-colorable and the inequality, $||V_i| - |V_j|| \le 1$ holds for every i and j.
Hence $\chi = (C_m \dot{\vee} P_n) = 4$ if $|m - n| \ge 3$.

3. $3 \le |m - n| \le \left(\lceil \frac{m}{2} \rceil + 1\right)$ for $m < n$

   The total number of vertices of the cycle and its subdivided vertices are $2m$ and the number of vertices of the path is n. Hence the total number of vertices is $2m + n$. Let us assign the colors equitably to the vertices $2m + n$. Assign color 1 to the vertices $v_i : 1 \le i \le \lfloor \frac{2m+n}{4} \rfloor$ of the cycle C_m.

   (i) If $\lfloor \frac{2m+n}{4} \rfloor = m$ then the remaining number of vertices will be $m + n$ (i.e.,) $\{u_1,\ u_3,\ u_5, \ldots, u_m\} \cup \{s_1,\ s_3,\ s_5, \ldots, s_n\}$. Since each vertex of the cycle is joined to every vertex of the path color 1 is forbidden. Also in the path graph since no two adjacent vertices can have same color we use two additional colors 2 and 3. To satisfy the equitability one more additional color should be used (i.e.,) color 4. Hence the colors 2, 3 and 4 are assigned, alternatively to the consecutive vertices $\{u_1,\ u_3,\ u_5, \ldots, u_m,\ s_1,\ s_3,\ s_5, \ldots, s_n\}$. Each of these three colors is assigned either $\lceil \frac{2m+n}{4} \rceil$ or $\lfloor \frac{2m+n}{4} \rfloor$ times.

   Hence all the four colors are assigned either $\lceil \frac{2m+n}{4} \rceil$ or $\lfloor \frac{2m+n}{4} \rfloor$ times such that the color classes differ in sizes by at most 1.

   (ii) If $\lfloor \frac{2m+n}{4} \rfloor \ne m$ then the remaining vertices of the cycle C_m are assigned color 2. Since each vertex of the cycle is joined to every vertex of the path colors 1 and 2 are forbidden. Also in the path graph since no two adjacent vertices can have same color we use two additional colors 3 and 4. Hence a minimum of four colors should be used. On checking the equitability maximum of four colors is enough. Color 2 is assigned to the subdivided vertices such that the total number of vertices having color 2 is either $\lceil \frac{2m+n}{4} \rceil$ or $\lfloor \frac{2m+n}{4} \rfloor$ times. Now for the remaining subdivided vertices and the vertices of the path colors 3 and 4 are assigned, alternatively. Hence all the four colors are assigned either $\lceil \frac{2m+n}{4} \rceil$ or $\lfloor \frac{2m+n}{4} \rfloor$ times such that the color classes differ in sizes by at most 1.

Therefore all the four colors are assigned either $\lceil \frac{2m+n}{4} \rceil$ or $\lfloor \frac{2m+n}{4} \rfloor$ times such that the color classes differ in sizes by at most 1.

Hence $\chi = (C_m \dot{\vee} P_n) = 4$ if $3 \leq |m-n| \geq \lceil \frac{m}{2} \rceil + 1$ for $m < n$.

4. $\lceil \frac{2m+n+1}{m+1} \rceil$ otherwise for $m < n$.

The total number of vertices of the subdivision vertex join graph of the cycle with path is $2m + n$. By the construction of equitable coloring, all the vertices receive the colors according to any one of the following cases:

(i) If all the vertices of the cycle receive the same color 1.

In this case, color 1 appears at the maximum of m times. Since each vertex of the cycle is connected to every vertices of the path, color 1 is forbidden. Hence the m subdivided vertices of the cycle and n vertices of the path should be given different colors. Number of colors to be assigned to these vertices is $\lceil \frac{m+n}{m+1} \rceil$.

Hence $\lceil \frac{m+n}{m+1} \rceil + 1 = \lceil \frac{2m+n+1}{m+1} \rceil$ colors are assigned to the vertices of the subdivision graph. Let color 2 be assigned to all the subdivided vertices of the cycle and the vertices of the path are assigned the remaining colors, alternatively.

(ii) If the vertices of the cycle receive the color 1 with another color 2.

In this case, color 1 appears at the maximum of m times. Since each vertex of the cycle is connected to every vertices of the path, color 1 is forbidden. Color 2 is assigned to the remaining vertices of the cycle. Hence colors 1 and 2 are forbidden for the vertices of the path. Color 2 is assigned to the subdivided vertices of the cycle such that color 2 appears at most m times. Remaining subdivided vertices of the cycle and n vertices of the path should be given different colors to get an equitable coloring. Total number of colors received by all the vertices of the subdivision vertex join graph is $\lceil \frac{m+n}{m+1} \rceil + 1 = \lceil \frac{2m+n+1}{m+1} \rceil$.

Hence $\chi = (C_m \dot{\vee} P_n) = \lceil \frac{2m+n+1}{m+1} \rceil$ otherwise for $m < n$. □

3 Conclusion

In this present paper, we have discussed the equitable coloring on subdivision vertex join of cycle C_m with path P_n. As a motivation this work can be extended to find the equitable chromatic number, equitable edge chromatic number and equitable total chromatic number for subdivision vertex join of different families of graphs also in general an two arbitrary graphs.

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