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Prime sequences

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Abstract: Primes are considered in three sequences, of which two are exclusive to specific primes. These sequences have the integers represented in the form *nR* where *R* is the right-end-digit of the prime and *n* represents the remaining left digits which are given by linear equations.
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1 Introduction

An unexpected bias in the distribution of consecutive primes [2] is clearly apparent in the right-end-digit (RED) considerations of their distributions [3-10]. Three main sequences of primes occur when presented in the form *nR*, where *R* is the RED of the prime and *n* represents the remaining left digits; for example, for the prime 177, $n = 17$ and $R = 7$. Thus RED-defined sequences are embedded in three principal formats:

$$n = 3t + 2 \tag{1.1}$$

$$n = 3t \tag{1.2}$$

$$n = 3t + 1 \tag{1.3}$$

The aim of this paper is to consider the non-randomness of these sequences of RED-defined primes; this is also illustrated with modular rings and integer sequence analysis [3–10].

2 The sequence $n = 3t + 2$

For this sequence, $R = 1$ or 7 will always yield a composite integer value, but a prime integer value is possible if their RED is 3 or 9 (see Table 1), which appear to have a high prime 'yield'.

n	2	5	8	11	14	17	20	23	26	29	32	35	38
t	0	1	2	3	4	5	6	7	8	9	10	11	12
***R* = 1**	21	51	81	111	141	171	201	231	261	291	321	351	381
type	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*
***R* = 7**	27	57	87	117	147	177	207	237	267	297	327	357	387
type	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*	*c*
***R* = 3**	23	53	84	113	143	173	203	233	263	293	323	353	383
type	*p*	*p*	*p*	*p*	*c*	*p*	*c*	*p*	*p*	*p*	*c*	*p*	*p*
***R* = 9**	29	59	89	119	149	179	209	239	269	299	329	359	389
type	*p*	*p*	*p*	*c*	*p*	*p*	*c*	*p*	*p*	*c*	*c*	*p*	*p*

Table 1. Primality for $n = 3t + 2$ (where p denotes prime, c denotes composite)

Many sequences can obviously be obtained from the t values in equation (1.1). For instance,

$$n = a + 21j \tag{2.1}$$

as illustrated in Table 2 for $a = 2 + 3i$, in which 3 and 9 REDs of primes are often both produced for a given n.

a	REDs of primes	Range	% of primes	Total % of primes
2	3	(23; 13883)	53	76
	9	(29; 13679)	52	
5	3	(53; 10343)	54	82
	9	(59; 10559)	52	
8	3	(83; 10163)	50	72
	9	(89; 10589)	50	
11	3	(113; 23003)	46	46
14	3	(143; 10433)	54	80
	9	(149; 8969)	48	
17	3	(173; 10463)	62	74
	9	(179; 10259)	60	
20	9	(419; 10709)	56	56
23	3	(23; 13883)	53	76
	9	(29; 13679)	52	
26	3	(53; 10343)	54	82
	9	(59; 10559)	52	

Table 2. Equation (2.1) with $a = 2 + 3i$

3 The sequence $n = 3t$

Only primes with REDs equal to 1 and 7 are produced, since nR can be divided by 3. Similar forms of imbedded sequences apply as for $(2 + 3t)$ (see Table 3). In this case, $a = 3i$ in equation (2.1).

a	REDs of primes	Range	% of primes	Total % of primes
0	1	(211; 10501)	48	48
3	1	(31; 10531)	56	78
	7	(37; 9907)	50	
6	1	(61; 10141)	68	78
	7	(67; 10567)	50	
9	7	(97; 10597)	48	48
12	1	(331; 9781)	54	78
	7	(127; 10627)	58	
15	1	(151; 10651)	46	74
	7	(157; 10657)	56	
18	1	(181; 9631)	52	80
	7	(397; 10687)	56	
21	1	(211; 10501)	48	48
24	1	(31; 10531)	56	78
	7	(37; 9907)	50	
27	1	(61; 10141)	68	78
	7	(67; 10567)	50	

Table 3. Equation (2.1) with $a = 3i$

4 The sequence $n = 3t + 1$

This sequence produces primes with REDs 1, 3, 7 and 9. The embedded sequences produce primes for only three of the REDs for a given a. Since $R = 5$ can never be prime (except for $n = 0$), the imbedded sequences can produce primes with all four REDs 1, 3, 7, 9 (Table 4).

RED that is always composite for $n = i + 3t$	Imbedded sequences	REDs forming primes	R	% of primes produced	Total % of primes
1	$n = 16 + 21j$ $t = 5 + 7i$	3, 7, 9	1	0	93
			3	59	
			7	54	
			9	44	
3	$n = 13 + 21j$ $t = 4 + 7i$	1, 7, 9	1	35	90
			3	0	
			7	53	
			9	58	
5	$n = 10 + 21j$ $t = 3 + 7i$	1, 3, 7, 9	1	73	95
			3	70	
			5	0	
			7	51	
			9	55	

(contd.)

RED that is always composite for $n = i + 3t$	Imbedded sequences	REDs forming primes	R	% of primes produced	Total % of primes
7	$n = 7 + 21j$ $t = 2 + 7i$	1, 3, 9	1	50	93
			3	51	
			7	0	
			9	52	
9	$n = 4 + 21j$ $t = 1 + 7i$	1, 3, 7	1	50	93
			3	35	
			7	52	
			9	0	

Table 4. High percentage of primes in imbedded sequences

5 'Large' primes

Since the sequences have genuine structural features they should be applicable independently of the size of the integers.

Some examples are set out in Tables 5 and 6 for what one might call large primes.

Prime	R	Major series	Imbedded series	Remarks
104395301	1	$1 + 3t$	$n = 10 + 21j$ $j = 497120$	Integers with this n & $R = 3$, 7 or 9 could be prime
179426111	1	$1 + 3t$	$n = 1 + 21j$ $j = 854410$	Integers with this n & $R = 3$, 7 or 9 could be prime
179425033	3	$1 + 3t$	$n = 19 + 21j$ $j = 854404$	Integers with this n & $R = 1$, 7 or 9 could be prime
179434483	3	$1 + 3t$	$n = 19 + 21j$ $j = 854404$	Integers with this n & $R = 7$ could be prime
179425177	7	$3t$	$n = 2 + 21j$ $j = 854405$	Integers with this n & $R = 1$ could be prime
179434487	7	$1 + 3t$	$n = 19 + 21j$ $j = 854449$	Integers with this n & $R = 3$, or 9 could be prime
179425319	9	$2 + 3t$	$n = 5 + 21j$ $j = 854406$	Integers with this n & $R = 3$ could be prime
179426369	9	$2 + 3t$	$n = 5 + 21j$ $j = 854411$	Integer with this n & $R = 3$ could be prime

Table 5. Imbedded series for 'large' primes

Prime	*R*	Major series	Imbedded series	Remarks
817504253838041641	1	$3t$	$n = 15 + 21j$ $j = 3892877399228769$	Integer with this n & $R = 7$ could be prime
961748941982451653	3	$2 + 3t$	$n = 2 + 21j$ $j = 4579956866587103$	Integer with this n & $R = 9$ could be prime
275604547295075147	7	$1 + 3t$	$n = 10 + 21j$ $j = 1312402606167024$	Integer with this n & $R = 1, 3, 9$ could be prime
593441861613651349	9	$1 + 3t$	$n = 1 + 21j$ $j = 2825913626731673$	Integer with this n & $R = 1, 3, 7$ could be prime

Table 6. Imbedded series for 'large' primes

6 Fibonacci and Mersenne primes

6.1 Fibonacci primes

When $R = 3$, composites always occur when $n = 2 + 3t$ for $p > 29$ and there is a bias towards primes for $R = 7$ (see Tables 7 and 8). Nine primes are formed from 25 p values (36%).

Of the 25 p values, twelve yield the Fibonacci number, F_p, [10] with $n = 1 + 3t$, six have $n = 2 + 3t$, and six have $n = 3t$ (see Table 7) [12].

When $n = 13 + 21j$ or $17 + 21j$, all F_p are composite (see Table 7), and all F_p with $R = 1$ are composite for the range in this table. For instance, $F_{13} = 1, 346, 269 = 557 \times 2417$: prime subscript but composite number, whereas $F_{29} = 514, 229$ which is prime: prime subscript and prime number. Similarly with $F_{13} = 233$. In this case, $R = 3$ as in Table 1, and $a = 2$ with a RED of 3 as in Table 2; that is, $a = n - 2j = 23 - 21$. The search for new near-patterns among primes and prime-indexed numbers goes on with a variety of interesting techniques [1].

p	F_p	*R*	$n = f(t)$	*j*	Type
7	13	3	$1 + 3t$	---	*p*
11	89	9	$2 + 3t$	0	*p*
13	233	3	$2 + 3t$	1	*p*
17	1597	7	$3t$	7	*p*
19	4181	1	$1 + 3t$	19	*c*
23	28657	7	$3t$	136	*p*
29	514229	9	$2 + 3t$	2448	*p*
31	1346269	9	$1 + 3t$	6410	*C*
37	24157817	7	$1 + 3t$	15037	*c*
41	165580141	1	$3t$	788476	*c*
43	433494437	7	$1 + 3t$	2064259	*p*
47	2971215073	3	$1 + 3t$	14148643	*p*
53	53316291173	3	$2 + 3t$	253887100	*c*
59	956722026041	1	$1 + 3t$	4555819171	*c*

(contd.)

p	F_p	R	$n = f(t)$	j	Type
61	2504730781961	1	$1 + 3t$	1927289438	c
67	44945570212853	3	$2 + 3t$	214026524823	c
71	308061521170129	9	$1 + 3t$	1466959624619	c
73	806515533049393	3	$1 + 3t$	3840550157378	c
79	14472334024676221	1	$3t$	6891587607982	c
83	99194853094755497	7	$1 + 3t$	472356443308359	p
89	1779979416004714189	9	$1 + 3t$	8476092457165305	c
97	83621143489848422977	7	$3t$	398319592138O230	c
101	573147844013817084101	1	$1 + 3t$	2729275447684843257	c
103	1500520536206896083277	7	$3t$	7145335886699505158	c
107	10284720757613717413913	3	$2 + 3t$	48974860750541511494	c

Table 7. Fibonacci types ($5 \leq F_p \leq 10284720757613717413913$)

R	$2 + 3t$	$3t$	$1 + 3t$	Total
1	–	–	–	0
3	1	–	2	3
7	–	2	2	4
9	2	–	–	2

Table 8. Numbers of R

6.2 Mersenne primes

The Mersenne numbers, $M_m = 2^m - 1$ (m odd) have been known for centuries with interest centred on which m yield primes [12]. These primes are found in the very large number domain, yet the REDs can only be 1 or 7 (2^p REDs can only be 2 or 8). Moreover, in the nR form n always has the form $3t$.

7 Final comments

The largest known primes (Table 9) are expressed in the form

$$p = 2^m - 1. \tag{6.1}$$

Since m for these cases falls in class $\bar{1}_4 \left(m = 4r_1 + 1\right)$, the RED of 2^m will be 2 [13] so that RED of $2^m - 1 = 1$.

No. of digits × 10^6	$m \in \bar{1}_4$	2^{4r_1+1}	RED of prime
22.3	74207281	2	1
17.4	51885161	2	1
13.0	43112609	2	1
13.0	42643801	2	1

Table 9. REDs of Mersenne primes

These primes can fall in ($n = 3t$) or ($n = 1 + 3t$) sequences, but $n \neq 16 + 21j$ (see Table 4).

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