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On two new combined 3-Fibonacci sequences

Krassimir T. Atanassov

Department of Bioinformatics and Mathematical Modelling
IBPhBME – Bulgarian Academy of Sciences
Acad. G. Bonchev Str. Bl. 105, Sofia-1113, Bulgaria
and
Intelligent Systems Laboratory
Prof. Asen Zlatarov University, Bourgas-8010, Bulgaria
e-mail: krat@bas.bg

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Abstract: Two new combined 3-Fibonacci sequences are introduced and the explicit formulae for their n-th members are given.
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1 Introduction

The author has introduced a series of extensions of the nature of the Fibonacci sequence (see, e.g., [1]). Here, two new combined 3-Fibonacci sequences are introduced.

2 First scheme

Let a, b, c, d be arbitrary real numbers. The first sequence has the form: $\alpha_0 = a, \beta_0 = b, \gamma_0 = c, \gamma_1 = d$ and for each natural number n:

$$\alpha_{n+2} = \gamma_{n+1} + \beta_{n+1},$$

$$\beta_{n+2} = \gamma_{n+1} + \alpha_{n+1},$$

$$\gamma_{n+2} = \gamma_{n+1} + \gamma_n.$$

The first values of sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are the following

n	α_n	γ_n	β_n
0	a		b
0		c	
1	$b+c$		$a+c$
1		d	
2	$a+c+d$		$b+c+d$
2		$c+d$	
3	$b+2c+2e$		$a+2c+2d$
3		$c+2d$	
4	$a+3c+4d$		$b+3c+4d$
4		$2c+3d$	
5	$b+5c+7d$		$a+5c+7d$
5		$3c+5d$	
6	$a+8c+12d$		$b+8c+12d$
6		$5c+8d$	
7	$b+13c+20d$		$a+13c+20d$
7		$8c+13d$	
8	$a+21c+33d$		$b+21c+33d$
8		$13c+21d$	
9	$b+34c+54d$		$a+34c+54d$
9		$21c+34d$	
	$\dots$	$\dots$	$\dots$

Let $\{F_n\}_{n=0}^{\infty}$ be the standard Fibonacci sequence, where for each natural number $n \geq 0$, $F_0 = 0, F_1 = 1$, and

$$F_{n+2} = F_{n+1} + F_n.$$

Theorem 1. For each natural number $n \geq 1$:

$$\alpha_{2n-1} = b + F_{2n-1}a + (F_{2n} - 1)d,$$

$$\alpha_{2n} = a + F_{2n}c + (F_{2n+1} - 1)d,$$

$$\beta_{2n-1} = a + F_{2n-1}c + (F_{2n} + 1)d,$$

$$\beta_{2n} = b + F_{2n}c + (F_{2n+1} - 1)d,$$

$$\gamma_{n+2} = F_{n+1}c + F_{n+2}d.$$

Proof: We can prove the Theorem, for example, by induction. For $n = 1$, the validity of the Theorem is checked directly from the above table. Let us assume that the Theorem is valid for some natural number $n \geq 1$. Then:

$$\begin{aligned}\alpha_{2n+1} &= \gamma_{2n} + \beta_{2n} \\ &= F_{2n-1}c + F_{2n}d + b + F_{2n}c + (F_{2n+1} - 1)d \\ &= b + (F_{2n-1} + F_{2n})c + (F_{2n} + F_{2n+1} - 1)d \\ &= b + F_{2n+1}c + (F_{2n+2} - 1)d.\end{aligned}$$

$$\begin{aligned}\alpha_{2n+2} &= \gamma_{2n+1} + \beta_{2n+1} \\ &= F_{2n}c + F_{2n+1}d + a + F_{2n+1}c + (F_{2n+2} - 1)d \\ &= a + (F_{2n} + F_{2n+1})c + (F_{2n+1} + F_{2n+2} - 1)d \\ &= b + F_{2n+2}c + (F_{2n+3} - 1)d.\end{aligned}$$

The rest formulas are checked by analogy. □

3 Second scheme

Let a, b, c, d be arbitrary real numbers. The second sequence has the form: $\alpha_0 = a, \beta_0 = b, \gamma_0 = c, \alpha_2 = d$ and for each natural number n:

$$\alpha_{n+1} = \alpha_{n+1} + \alpha_n,$$

$$\beta_{n+1} = \alpha_{n+1} + \gamma_n,$$

$$\gamma_{n+1} = \alpha_{n+1} + \beta_n.$$

The first values of sequences $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are the following

n	β_n	α_n	γ_n
0		a	
0	b		c
1		d	
1	$c+d$		$b+d$
2		$a+d$	
2	$a+b+2d$		$a+c+2d$
3		$a+2d$	
3	$2a+c+4d$		$2a+b+4d$
4		$2a+3d$	
4	$4a+b+7d$		$4a+c+7d$
5		$3a+5d$	
5	$7a+c+12d$		$7a+b+12d$
6		$5a+8d$	
6	$12a+b+20d$		$12a+c+20d$
7		$8a+13d$	
7	$20a+c+33d$		$20a+b+33d$
8		$13a+21d$	
8	$33a+b+54d$		$33a+c+54d$
9		$21a+34d$	
9	$54a+c+88d$		$54a+b+88d$
	$\ldots$	$\ldots$	$\ldots$

Theorem 2. For each natural number $n \geq 1$:

$$\alpha_n = F_{n-1}c + F_n d,$$
$$\beta_{2n-1} = (F_{2n} - 1)a + b + (F_{2n+1} - 1)d,$$
$$\beta_{2n} = (F_{2n+1} - 1)a + c + (F_{2n+2} - 1)d,$$
$$\gamma_{2n-1} = (F_{2n} - 1)a + c + (F_{2n+1} - 1)d,$$
$$\gamma_{2n} = (F_{2n+1} - 1)a + b + (F_{2n+2} - 1)d.$$

Proof: The proof is similar. □

Another new scheme, modifying the standard form of 2-Fibonacci and 3-Fibonacci sequences and the above two sequences, will be discussed in near future.

References

[1] Atanassov K., Atanassova, V., Shannon, A., & Turner, J. (2002) *New Visual Perspectives on Fibonacci Numbers*, World Scientific, New Jersey.