Notes on Number Theory and Discrete Mathematics
Print ISSN 1310–5132, Online ISSN 2367–8275
Vol. 24, 2018, No. 2, 74–84
DOI: 10.7546/nntdm.2018.24.2.74-84

Global equitable domination in some degree splitting graphs

S. K. Vaidya[1] and R. M. Pandit[2]

[1] Department of Mathematics, Saurashtra University
Rajkot, Gujarat, India
e-mail: samirkvaidya@yahoo.co.in

[2] Department of Mathematics, Government Polytechnic
Jamnagar, Gujarat, India
e-mail: pandit.rajesh@ymail.com

Received: 25 March 2016 **Accepted:** 7 May 2018

Abstract: A dominating set is called a global dominating set if it is a dominating set of a graph G and its complement $\overline{G}$. A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G) - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|d_G(u) - d_G(v)| \leq 1$. An equitable dominating set D of a graph G is a global equitable dominating set if it is also an equitable dominating set of the complement of G. The minimum cardinality of a global equitable dominating set of G is called the global equitable domination number of G which is denoted by $\gamma_g^e(G)$. We explore this concept in the context of degree splitting graphs of some graphs.
Keywords: Equitable dominating set, Global equitable dominating set, Global equitable domination number, Degree splitting graph.
2010 Mathematics Subject Classification: 05C69.

1 Introduction

The domination in graphs is one of the concepts in graph theory which has attracted many researchers to work on it. An excellent survey on the concept of domination and its related parameters can be found in Haynes *et al.* [4] while some advanced topics on domination are explored by the same authors in [5]. The concept of domination has many interesting applications in the study of social networks which has motivated Prof. E. Sampathkumar to introduce the concept of equitable domination in graphs.

Let G be a graph of road network linking various locations. It is desirable to maintain the supply to these locations uninterruptedly by using the alternative links even if the original links get disturbed. Then the problem of finding the minimum number of supplying stations needed to accomplish this task is equivalent to find the global domination number. The concept of global domination was introduced by Sampathkumar [10].

Many domination models are introduced by combining two different domination parameters. Independent domination, connected domination, total global domination, equitable domination, global equitable domination are just to name a few. Motivated through the concepts of global domination and equitable domination, a new concept of global equitable domination was conceived by Basavanagoud and Teli [2] and formalized as well as explored by Vaidya and Pandit [13, 14].

We begin with a simple, finite and undirected graph G with the vertex set $V(G)$ and the edge set $E(G)$. The set $D \subseteq V(G)$ is called a dominating set if every vertex $v \in V(G)$ is either an element of D or is adjacent to an element of D. The minimum cardinality of a dominating set of G is called the domination number of G which is denoted by $\gamma(G)$.

The complement $\overline{G}$ of G is the graph with vertex set $V(G)$ in which two vertices are adjacent in $\overline{G}$ if they are not adjacent in G. We denote the degree of a vertex v in G by $d_G(v)$. A vertex of degree one is called a pendant vertex.

A dominating set D of G is called a global dominating set if it is also a dominating set of $\overline{G}$. The global domination number $\gamma_g(G)$ is the minimum cardinality of a global dominating set of G. Many researchers have explored this concept. For example, Gangadharappa and Desai [3] have discussed the global domination in graphs of small diameters while Vaidya and Pandit [12] have investigated the global domination number of the larger graphs obtained by some graph operations on a given graph. Kulli and Janakiram [6] have introduced the concept of total global dominating sets.

A subset D of $V(G)$ is called an equitable dominating set if for every $v \in V(G) - D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|d_G(u) - d_G(v)| \leq 1$. The minimum cardinality of such a dominating set is called the equitable domination number of G which is denoted by $\gamma^e(G)$. Swaminathan and Dharmalingam [11] studied the equitable domination in graphs and characterized the minimal equitable dominating sets. Murugan and Emmanuel [7] identified the relationship among domination, equitable domination and independent domination in graphs while Revathi and Harinarayanan [9] studied the equitable domination in fuzzy graphs.

A vertex $u \in V(G)$ is called an equitable isolate if $|d_G(u) - d_G(v)| \geq 2$ for all $v \in N(u)$. An equitable isolate must belong to any equitable dominating set of G. Clearly, the isolated vertices are the equitable isolates. Hence, $I_s \subseteq I_e \subseteq D$ for every equitable dominating set D where I_s and I_e denote the sets of all isolated vertices and all equitable isolates of G, respectively.

A subset D of $V(G)$ is called a global equitable dominating set if D is an equitable dominating set of both G and $\overline{G}$. The minimum cardinality of a global equitable dominating set of G is called the global equitable domination number of G and it is denoted by $\gamma_g^e(G)$.

Since at least two vertices are required to equitably dominate both G and $\overline{G}$, we have

$2 \leq \gamma_g^e(G) \leq n$ for every graph of order $n > 1$. Both of these bounds are sharp. In particular, the equality of the lower bound is attained by $P_n\,(2 \leq n \leq 6)$ and $K_{m,n}\,(\,|m-n| \leq 1\,)$ while the upper bound is achieved by K_n, $K_{1,n}$ and $K_{m,n}\,(\,|m-n| \geq 2\,)$.

For any real number n, $\lceil n \rceil$ denotes the smallest integer not less than n and $\lfloor n \rfloor$ denotes the greatest integer not greater than n.

For notations and graph theoretic terminology not defined herein, we refer the readers to West [15] while the terms related to the concept of domination are used in the sense of Haynes *et al.* [4].

In [8], R. Ponaraj and S. Somasundaram have initiated a study of degree splitting graph of a graph which is stated as follows:

Let $G = (V(G), E(G))$ be a graph with $V(G) = S_1 \bigcup S_2 \bigcup \ldots \bigcup S_t \bigcup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V(G) - \bigcup S_i$. Note that if $V(G) = \bigcup S_i$ then $T = \emptyset$.

The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, \ldots, w_t$ and joining w_i to each vertex of $S_i\,(1 \leq i \leq t)$.

The domination in degree splitting graphs is discussed in [1]. In this paper, we determine the exact values of the global equitable domination number of degree splitting graphs of path P_n, shell S_n, bistar $B_{n,n}$, gear graph G_n, helm H_n, closed helm CH_n and flower graph Fl_n.
In Figure 1, a graph G and its degree splitting graph $DS(G)$ are shown.

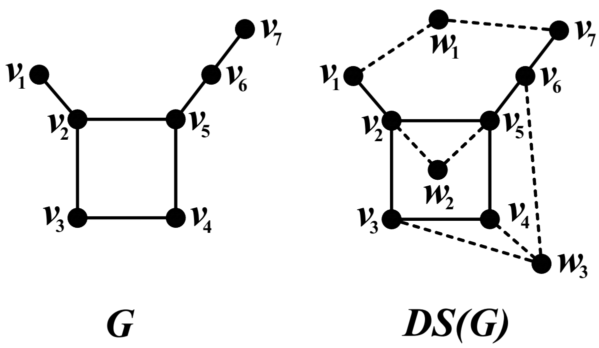

Figure 1. Graph G and its degree splitting graph $DS(G)$

Here, the vertices of G can be categorized into three types based on their degrees. Thus,

- S_1 = set of vertices of degree $1 = \{v_1, v_7\}$,
- S_2 = set of vertices of degree $3 = \{v_2, v_5\}$,
- S_3 = set of vertices of degree $2 = \{v_3, v_4, v_6\}$ and $T = \emptyset$.

Therefore, in order to construct $DS(G)$ from G, three vertices w_1, w_2, w_3 must be added in G corresponding to the sets S_1, S_2, S_3, respectively. In Figure 1, the dotted edges in $DS(G)$ represent the newly added edges.

2 Main results

Proposition 2.1. [2]
(i) For the path $P_n\,(n \geq 4)$, $\gamma_g^e(P_n) = \lceil \frac{n}{3} \rceil$.

(ii) For the cycle C_n, $\gamma_g^e(C_n) = \begin{cases} 3 & \text{if } n = 3, 5 \\ \lceil \frac{n}{3} \rceil & \text{otherwise.} \end{cases}$

Theorem 2.2. For the path P_n,

$$\gamma_g^e(DS(P_n)) = \begin{cases} 3 & \text{if } n = 2 \\ \lceil \frac{n+1}{3} \rceil & \text{if } 3 \leq n \leq 6 \\ \lceil \frac{n+4}{3} \rceil & \text{if } n \geq 7. \end{cases}$$

Proof: Let v_1 and v_n be the pendant vertices of path P_n and let $v_2, v_3, \ldots, v_{n-1}$ be the internal vertices of P_n which are of degree two. Hence, for $n \geq 4$, let $V(P_n) = S_1 \bigcup S_2$ where $S_1 = \{v_1, v_n\}$ and $S_2 = \{v_2, v_3, \ldots, v_{n-1}\}$. In order to obtain $DS(P_n)$ from $P_n\,(n \geq 4)$, we add two new vertices w_1, w_2 corresponding to S_1, S_2, respectively, and join w_i to each vertex of S_i. Then $V(DS(P_n)) = V(P_n) \bigcup \{w_1, w_2\}$. Hence, $|V(DS(P_n))| = n + 2$. For $n = 2, 3$, $V(DS(P_n)) = V(P_n) \bigcup \{w_1\}$.

Since $DS(P_2)$ is isomorphic to the cycle C_3, we have $\gamma_g^e(DS(P_2)) = \gamma_g^e(C_3) = 3$. Now, w_2 is an equitable isolate in $\overline{DS(P_6)}$. From the nature of a graph, the following set D is clearly a global equitable dominating set of $DS(P_n)$ with minimum cardinality.

$$D = \begin{cases} \{w_1, v_1\} & \text{for } \; n = 3 \\ \{w_1, w_2\} & \text{for } \; n = 4, 5 \\ \{w_1, w_2, v_1\} & \text{for } \; n = 6. \end{cases}$$

Hence,

$$\gamma_g^e(DS(P_n)) = \begin{cases} 3 & \text{if } n = 2 \\ \lceil \frac{n+1}{3} \rceil & \text{if } 3 \leq n \leq 6. \end{cases}$$

For $n \geq 7$, the vertex w_2 is the equitable isolate in both $DS(P_n)$ and $\overline{DS(P_n)}$. Therefore, every global equitable dominating set of $DS(P_n)$ must contain w_2. Now, the vertex w_1 equitably dominates the pendant vertices of P_n. Moreover, the remaining vertices of $DS(P_n)$ induce a subgraph P_{n-2} and by Proposition 2.1, $\gamma_g^e(P_{n-2}) = \lceil \frac{n-2}{3} \rceil$. This implies that every equitable dominating set of $DS(P_n)$ must have at least $\lceil \frac{n-2}{3} \rceil + 1 + 1 = \lceil \frac{n+4}{3} \rceil$ vertices. Furthermore, any two internal vertices u and v of P_n with $d(u, v) \geq 3$ are enough to equitably dominate all the vertices of $\overline{DS(P_n)}$ and every equitable dominating set of $DS(P_n)$ must have such vertices. Hence, for every global equitable dominating set S of $DS(P_n)$, $|S| \geq \lceil \frac{n+4}{3} \rceil$ which implies that $\gamma_g^e(DS(P_n)) = \lceil \frac{n+4}{3} \rceil$ for $n \geq 7$.

Thus, we have

$$\gamma_g^e(DS(P_n)) = \begin{cases} 3 & \text{if } n = 2 \\ \lceil \frac{n+1}{3} \rceil & \text{if } 3 \leq n \leq 6 \\ \lceil \frac{n+4}{3} \rceil & \text{if } n \geq 7. \end{cases}$$

Hence, the theorem is proved. □

Definition 2.3. A shell S_n $(n > 3)$ is the graph obtained by taking $(n-3)$ concurrent chords in a cycle C_n. The vertex at which all the chords are concurrent is called the apex. The shell S_n is also called fan f_{n-1}. That is, $S_n = f_{n-1} = P_{n-1} + K_1$.

Theorem 2.4. For the shell S_n,

$$\gamma_g^e(DS(S_n)) = \begin{cases} 4 & \text{if } n = 7 \\ \lceil \frac{n+6}{3} \rceil & \text{otherwise.} \end{cases}$$

Proof: Let S_n be the shell with the vertex set $\{c, v_1, v_2, \ldots, v_{n-1}\}$ where c denotes the apex vertex of S_n.

There are three types of vertices in S_n $(n \neq 4)$:
(i) The vertices of degree 2 namely v_1, v_{n-1}.
(ii) The vertices of degree 3 namely $v_2, v_3, \ldots, v_{n-2}$.
(iii) The apex vertex of degree $n-1$ namely c.

Thus, let $V(S_n) = S_1 \bigcup S_2 \bigcup T$ where $S_1 = \{v_1, v_{n-1}\}$, $S_2 = \{v_2, v_3, \ldots, v_{n-2}\}$ and $T = \{c\}$. Hence, we add two new vertices w_1, w_2 corresponding to S_1, S_2 respectively to obtain $DS(S_n)$ from S_n. Then $V(DS(S_n)) = V(S_n) \bigcup \{w_1, w_2\}$ and $|V(DS(S_n)| = n + 2$.

For $n \leq 8$, the vertex c is an equitable isolate in $\overline{DS(S_5)}, \overline{DS(S_6)}, DS(S_7)$ and $DS(S_8)$ while the vertex w_2 is an equitable isolate in $DS(S_4)$ and $DS(S_5)$. Moreover, the vertices c, v_2 and c, w_1 are equitable isolates in $\overline{DS(S_4)}$ and $\overline{DS(S_7)}$, respectively, while the vertices c, w_1, w_2 are equitable isolates in $\overline{DS(S_8)}$. Since an equitable isolate of G must belong to every equitable dominating set of G and from the nature of a graph, it follows that the following set D is a global equitable dominating set of $DS(S_n)$ with minimum cardinality.

$$D = \begin{cases} \{c, w_1, w_2, v_2\} & \text{for } \quad n = 4 \\ \{c, w_2, v_1, v_4\} & \text{for } \quad n = 5, 6 \\ \{c, w_1, v_3, v_4\} & \text{for } \quad n = 7 \\ \{c, w_1, w_2, v_1, v_2\} & \text{for } \quad n = 8. \end{cases}$$

Thus, $\gamma_g^e(DS(S_7)) = 4$ and $\gamma_g^e(DS(S_n)) = \lceil \frac{n+6}{3} \rceil$ for $n \leq 8$, $n \neq 7$.

For $n \geq 9$, the vertices c, w_2 and c, w_1, w_2 are the equitable isolates in $DS(S_n)$ and $\overline{DS(S_n)}$, respectively. Therefore, these equitable isolates must belong to every global equitable dominating set of $DS(S_n)$. Moreover, the vertices in $V(DS(S_n)) - \{c, w_2\}$ induce the cycle C_n as a subgraph and by Proposition 2.1, $\gamma_g^e(C_n) = \lceil \frac{n}{3} \rceil$. Hence, at least $\lceil \frac{n}{3} \rceil + 2 = \lceil \frac{n+6}{3} \rceil$ vertices are required to equitably dominate all the vertices of $DS(S_n)$ implying that $\gamma^e(DS(S_n)) = \lceil \frac{n+6}{3} \rceil$. Now, any two vertices $u, v \in V(DS(S_n)) - \{c, w_1, w_2\}$ with $d(u, v) \geq 3$ are enough to equitably dominate all the vertices in $V(\overline{DS(S_n)}) - \{c, w_1, w_2\}$ and every equitable dominating set of $DS(S_n)$ must contain such vertices. This implies that, $\gamma_g^e(DS(S_n)) = \gamma^e(DS(S_n)) = \lceil \frac{n+6}{3} \rceil$.

Thus,

$$\gamma_g^e(DS(S_n)) = \begin{cases} 4 & \text{if } n = 7 \\ \lceil \frac{n+6}{3} \rceil & \text{otherwise.} \end{cases}$$

This completes the proof. □

Definition 2.5. The bistar $B_{n,n}$ is a graph obtained by joining the apex vertices of two copies of $K_{1,n}$ by an edge.

Theorem 2.6. For the bistar $B_{n,n}$,

$$\gamma_g^e(DS(B_{n,n})) = 2(n+2).$$

Proof: Let u_i, v_i $(1 \leq i \leq n)$ be the pendant vertices of $B_{n,n}$ and let c_1, c_2 denote the apex vertices of two copies of $K_{1,n}$. There are two types of vertices in $B_{n,n}$ namely, u_i, v_i $(1 \leq i \leq n)$ of degree 1 and c_1, c_2 of degree $n+1$. Thus, let $V(B_{n,n}) = S_1 \bigcup S_2$ where $S_1 = \{u_i, v_i : 1 \leq i \leq n\}$ and $S_2 = \{c_1, c_2\}$. In order to obtain $DS(B_{n,n})$ from $B_{n,n}$, we add two new vertices w_1, w_2 corresponding to S_1, S_2, respectively, and join w_i to each vertex of S_i. Hence, $V(DS(B_{n,n})) = V(B_{n,n}) \bigcup \{w_1, w_2\}$ and $|V(DS(B_{n,n}))| = 2(n+2)$.

Now, an equitable isolate in G must belong to every equitable dominating set of G. Since all the vertices of $DS(B_{n,n})$ are equitable isolates, it follows that $V(DS(B_{n,n}))$ is the only equitable dominating set of $DS(B_{n,n})$. Hence, $\gamma_g^e(DS(B_{n,n})) = |V(DS(B_{n,n}))| = 2(n+2)$. □

Definition 2.7. A gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the $(n-1)$-st cycle of W_n.

Theorem 2.8. For the gear graph G_n,

$$\gamma_g^e(DS(G_n)) = \begin{cases} \lceil \frac{n}{2} \rceil & \text{if } n = 4, 5 \\ 5 & \text{if } n = 6 \\ \lceil \frac{2n+7}{3} \rceil & \text{otherwise.} \end{cases}$$

Proof: Let c denote the apex vertex of wheel W_n and let $v_1, v_2, \ldots, v_{n-1}$ be the rim vertices of W_n. To obtain the gear graph G_n, subdivide each of the rim edges of W_n by the vertices $u_1, u_2, \ldots, u_{n-1}$ where each u_i is added between v_i and v_{i+1} for $i = 1, 2, \ldots, n-2$ and u_{n-1} is added between v_1 and v_{n-1}.

There are three types of vertices in G_n $(n \neq 4)$:

(i) The vertices of degree 3 namely $v_1, v_2, \ldots, v_{n-1}$.

(ii) The vertices of degree 2 namely $u_1, u_2, \ldots, u_{n-1}$.

(iii) A vertex of degree $n-1$ namely c.

Thus, let $V(G_n) = S_1 \bigcup S_2 \bigcup T$ where $S_1 = \{v_1, v_2, \ldots, v_{n-1}\}$, $S_2 = \{u_1, u_2, \ldots, u_{n-1}\}$ and $T = \{c\}$. Hence, we add two new vertices w_1, w_2 corresponding to S_1, S_2 respectively to obtain $DS(G_n)$ from G_n. Then $V(DS(G_n)) = V(G_n) \bigcup \{w_1, w_2\}$ and $|V(DS(G_n))| = 2n + 1$.

Now, the sets $D_1 = \{w_1, w_2\}$ and $D_2 = \{w_1, w_2, c\}$ are the global equitable dominating sets of $DS(G_4)$ and $DS(G_5)$ respectively with minimum cardinality. Hence, $\gamma_g^e(DS(G_n)) = \lceil \frac{n}{2} \rceil$ for $n = 4, 5$.

For $n = 6$, the vertex w_2 is an equitable isolate in $DS(G_6)$ and the set $D = \{w_2, v_1, v_2, v_5, u_3\}$ is clearly a global equitable dominating set with minimum cardinality. Hence, $\gamma_g^e(DS(G_6)) = 5$.

For $n \geq 7$, the vertices c, w_1 and w_2 are the equitable isolates in $DS(G_n)$. Therefore, every global equitable dominating set of $DS(G_n)$ must contain these equitable isolates. Now, the remaining vertices of $DS(G_n)$ induce the cycle $C_{2(n-1)}$ as a subgraph and by Proposition 2.1, $\gamma_g^e(C_{2(n-1)}) = \left\lceil \frac{2(n-1)}{3} \right\rceil$. Hence, $\gamma_g^e(DS(G_n)) = \left\lceil \frac{2(n-1)}{3} \right\rceil + 3 = \lceil \frac{2n+7}{3} \rceil$.

Thus, we have

$$\gamma_g^e(DS(G_n)) = \begin{cases} \lceil \frac{n}{2} \rceil & \text{if } n = 4, 5 \\ 5 & \text{if } n = 6 \\ \lceil \frac{2n+7}{3} \rceil & \text{otherwise.} \end{cases}$$

This completes the proof. □

Definition 2.9. The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each of its rim vertices.

Theorem 2.10. For the helm H_n,

$$\gamma_g^e(DS(H_n)) = \begin{cases} n + 2 & \text{if } n = 4, 6, 7 \\ 6 & \text{if } n = 5 \\ \lceil \frac{4n+5}{3} \rceil & \text{if } n \geq 8. \end{cases}$$

Proof: Let $v_1, v_2, \ldots, v_{n-1}$ be the rim vertices of wheel W_n and let c denote the apex vertex of the helm H_n. Let $u_1, u_2, \ldots, u_{n-1}$ be the pendant vertices of the helm H_n.
There are three types of vertices in H_n $(n \neq 5)$:
(i) The vertices of degree 4 namely $v_1, v_2, \ldots, v_{n-1}$.
(ii) The vertices of degree 1 namely $u_1, u_2, \ldots, u_{n-1}$.
(iii) A vertex of degree $n - 1$ namely c.

Thus, let $V(H_n) = S_1 \bigcup S_2 \bigcup T$ where $S_1 = \{v_1, v_2, \ldots, v_{n-1}\}$, $S_2 = \{u_1, u_2, \ldots, u_{n-1}\}$ and $T = \{c\}$. In order to obtain $DS(H_n)$ from H_n, we add two new vertices w_1, w_2 corresponding to S_1, S_2, respectively. Then $V(DS(H_n)) = V(H_n) \bigcup \{w_1, w_2\}$ and $|V(DS(H_n))| = 2n + 1$.

Now, the vertices c, w_1 are equitable isolates in $DS(H_4)$ while the vertices v_1, v_2 and v_3 are equitable isolates $\overline{DS(H_4)}$. Moreover, the vertices $w_2, u_1, u_2, \ldots, u_{n-1}$ are equitable isolates in $DS(H_n)$ for $n = 5, 6, 7$. Since an equitable isolate in G must belong to every global equitable dominating set of G, the following set D is clearly a global equitable dominating set of $DS(H_n)$ with minimum cardinality.

$$D = \begin{cases} \{c, w_1, w_2, v_1, v_2, v_3\} & \text{for} \quad n = 4 \\ \{w_1, w_2, u_1, u_2, u_3, u_4\} & \text{for} \quad n = 5 \\ \{c, w_1, w_2, u_1, u_2, \ldots, u_{n-1}\} & \text{for} \quad n = 6, 7. \end{cases}$$

Thus, $\gamma_g^e(DS(H_5)) = 6$ and $\gamma_g^e(DS(H_n)) = n + 2$ if $n = 4, 6, 7$.

For $n \geq 8$, the vertices $c, w_1, w_2, u_1, u_2, \ldots, u_{n-1}$ are the equitable isolates in $DS(H_n)$. Hence, these vertices must belong to every global equitable dominating set of $DS(H_n)$. Moreover, the remaining vertices of $DS(H_n)$ induce the cycle C_{n-1} as a subgraph and by Proposition 2.1, $\gamma_g^e(C_{n-1}) = \lceil \frac{n-1}{3} \rceil$. Therefore, at least $\lceil \frac{n-1}{3} \rceil + n + 2 = \lceil \frac{4n+5}{3} \rceil$ vertices are required to equitably dominate both $DS(H_n)$ and $\overline{DS(H_n)}$. This implies that $\gamma_g^e(DS(H_n)) = \lceil \frac{4n+5}{3} \rceil$ for $n \geq 8$.

Thus,

$$\gamma_g^e(DS(H_n)) = \begin{cases} n + 2 & \text{if } n = 4, 6, 7 \\ 6 & \text{if } n = 5 \\ \lceil \frac{4n+5}{3} \rceil & \text{if } n \geq 8. \end{cases}$$

This completes the proof. □

Definition 2.11. The closed helm CH_n is the graph obtained from a helm by joining each pendant vertex to form a cycle.

Theorem 2.12. [13] For the closed helm CH_n $(n > 5)$,

$$\gamma_g^e(CH_n) = \begin{cases} \lfloor \frac{n+2}{2} \rfloor & \text{if n} \equiv 1 \pmod 4 \\ \lceil \frac{n+2}{2} \rceil & \text{if n} \equiv 0,\ 2\ or\ 3 \pmod 4. \end{cases}$$

Theorem 2.13. For the closed helm CH_n,

$$\gamma_g^e(DS(CH_n)) = \begin{cases} \lceil \frac{n+2}{2} \rceil & \text{if } n = 4, 7 \\ \lfloor \frac{n}{2} \rfloor & \text{if } n = 5, 6. \end{cases}$$

and for $n \geq 8$,

$$\gamma_g^e(DS(CH_n)) = \begin{cases} \lfloor \frac{n+6}{2} \rfloor & \text{if n} \equiv 1 \pmod 4 \\ \lceil \frac{n+6}{2} \rceil & \text{if n} \equiv 0,\ 2\ or\ 3 \pmod 4. \end{cases}$$

Proof: Let c denote the apex vertex of closed helm CH_n.
There are three types of vertices in CH_n $(n > 5)$:
(i) The vertices of degree 4 namely $v_1, v_2, \ldots, v_{n-1}$.
(ii) The vertices of degree 3 namely $u_1, u_2, \ldots, u_{n-1}$.
(iii) A vertex of degree $n - 1$ namely c.

Thus, for $n > 5$, let $V(CH_n) = S_1 \bigcup S_2 \bigcup T$, where $S_1 = \{v_1, v_2, \ldots, v_{n-1}\}$, $S_2 = \{u_1, u_2, u_3, \ldots, u_{n-1}\}$ and $T = \{c\}$. For $n = 4$, we take $S_1 = \{v_1, v_2, v_3\}$, $S_2 = \{c, u_1, u_2, u_3\}$ while for $n = 5$, we take $S_1 = \{c, v_1, v_2, v_3, v_4\}$ and $S_2 = \{u_1, u_2, u_3, u_4\}$. Hence, we add two new vertices w_1, w_2 corresponding to S_1, S_2 respectively to obtain $DS(CH_n)$ from CH_n. Then $V(DS(CH_n)) = V(CH_n) \bigcup \{w_1, w_2\}$ and $|V(DS(CH_n))| = 2n + 1$.

Now, the vertices w_2 and w_1 are the equitable isolates in $DS(CH_4)$ and $DS(CH_7)$, respectively. The sets $D_1 = \{w_1, w_2\}$ and $D_2 = \{c, w_1, w_2, u_2, u_5\}$ are clearly the global equitable dominating sets of $DS(CH_5)$ and $DS(CH_7)$ respectively with minimum cardinality while the set $D_3 = \{c, w_1, w_2\}$ is a global equitable dominating set of $DS(CH_4)$ as well as of $DS(CH_6)$ with minimum cardinality.

Hence,

$$\gamma_g^e(DS(CH_n)) = \begin{cases} \lceil \frac{n+2}{2} \rceil & \text{if } n = 4, 7 \\ \\ \lfloor \frac{n}{2} \rfloor & \text{if } n = 5, 6. \end{cases}$$

For $n \geq 8$, the vertices c, w_1, w_2 are equitable isolates in $DS(CH_n)$. Therefore, every global equitable dominating set of $DS(CH_n)$ must contain these equitable isolates. Now, the set $V(DS(CH_n)) - \{w_1, w_2\}$ induces the closed helm CH_n and the vertex c is also an equitable isolate in CH_n. Hence, at least $\gamma_g^e(CH_n) + 2$ vertices are required to equitably dominate both $DS(CH_n)$ and $\overline{DS(CH_n)}$. This implies that $\gamma_g^e(DS(CH_n)) = \gamma_g^e(CH_n) + 2$.

Moreover, by Theorem 2.12.,

$$\gamma_g^e(CH_n) = \begin{cases} \lfloor \frac{n+2}{2} \rfloor & \text{if n} \equiv 1 \pmod 4 \\ \\ \lceil \frac{n+2}{2} \rceil & \text{if n} \equiv 0,\ 2\ \mathit{or}\ 3\ \pmod 4. \end{cases}$$

Thus, for $n \geq 8$,

$$\gamma_g^e(DS(CH_n)) = \begin{cases} \lfloor \frac{n+6}{2} \rfloor & \text{if n} \equiv 1 \pmod 4 \\ \\ \lceil \frac{n+6}{2} \rceil & \text{if n} \equiv 0,\ 2\ \mathit{or}\ 3\ \pmod 4. \end{cases}$$

This completes the proof. □

Definition 2.14. The flower graph Fl_n is the graph obtained from the helm H_n by joining each pendant vertex to the apex vertex of the helm H_n.

Theorem 2.15. For the flower graph Fl_n,

$$\gamma_g^e(DS(Fl_n)) = \begin{cases} n + 2 & \text{if } n = 4, 6, 7 \\ \\ 3 & \text{if } n = 5 \\ \\ \lceil \frac{4n+5}{3} \rceil & \text{if } n \geq 8. \end{cases}$$

Proof: Let c denote the apex vertex of the flower graph Fl_n.
There are three types of vertices in Fl_n:

(i) The vertices of degree 4 namely $v_1, v_2, \ldots, v_{n-1}$.
(ii) The vertices of degree 2 namely $u_1, u_2, \ldots, u_{n-1}$.
(iii) A vertex of degree $2(n-1)$ namely c.

Hence, let $V(Fl_n) = S_1 \bigcup S_2 \bigcup T$ where $S_1 = \{v_1, v_2, \ldots, v_{n-1}\}$, $S_2 = \{u_1, u_2, \ldots, u_{n-1}\}$ and $T = \{c\}$. In order to obtain $DS(Fl_n)$ from Fl_n, we add two new vertices w_1, w_2 corresponding to S_1, S_2, respectively.

Then $V(DS(Fl_n)) = V(Fl_n) \bigcup \{w_1, w_2\}$ and $|V(DS(Fl_n))| = 2n+1$.

In Fl_4, the vertex w_1 and the vertices c, v_1, v_2, v_3 are equitable isolates in $DS(Fl_4)$ and $\overline{DS(Fl_4)}$, respectively. Now, an equitable isolate in G must belong to every global equitable dominating set of G. Hence, $D = \{c, v_1, v_2, v_3, w_1, w_2\}$ is clearly a global equitable dominating set with minimum cardinality. This implies that $\gamma_g^e(DS(Fl_4)) = 6$.

In Fl_5, the apex vertex c is the equitable isolate in both $DS(Fl_5)$ and $\overline{DS(Fl_5)}$. Now, there is no vertex in $DS(Fl_5)$ which can equitably dominate all the vertices of $DS(Fl_5)$ except the vertex c. Hence, for any global equitable dominating set S, $|S| > 2$. But, $D = \{c, w_1, w_2\}$ is a global equitable dominating set of $DS(Fl_5)$ with $|D| = 3$. Thus, $\gamma_g^e(DS(Fl_5)) = 3$.

For $n = 6, 7$, the vertices $c, w_2, u_1, u_2, \ldots, u_{n-1}$ are the equitable isolates in $DS(Fl_n)$ while c is the equitable isolate in $\overline{DS(Fl_n)}$. Therefore, they must belong to every global equitable dominating set of $DS(Fl_n)$. Hence, the set $D = \{c, w_1, w_2, u_1, u_2, \ldots, u_{n-1}\}$ is clearly a global equitable dominating set of $DS(Fl_n)$ with minimum cardinality. Thus, $\gamma_g^e(DS(Fl_n)) = n+2$ for $n = 6, 7$.

For $n \geq 8$, the vertex c is the equitable isolate in both $DS(Fl_n)$ and $\overline{DS(Fl_n)}$. Therefore, every global equitable dominating set of $DS(Fl_n)$ must contain c. Moreover, as the vertices $w_1, w_2, u_1, u_2, \ldots, u_{n-1}$ are equitable isolates in $DS(Fl_n)$, they must belong to every global equitable dominating set of $DS(Fl_n)$. Furthermore, the remaining vertices of $DS(Fl_n)$ induce the cycle C_{n-1} as a subgraph and by Proposition 2.1, $\gamma_g^e(C_{n-1}) = \lceil \frac{n-1}{3} \rceil$. Therefore, at least $n+2+\lceil \frac{n-1}{3} \rceil = \lceil \frac{4n+5}{3} \rceil$ vertices are required to equitably dominate both $DS(Fl_n)$ and $\overline{DS(Fl_n)}$. Hence, $\gamma_g^e(DS(Fl_n)) = \lceil \frac{4n+5}{3} \rceil$.

Thus,

$$\gamma_g^e(DS(Fl_n)) = \begin{cases} n+2 & \text{if } n = 4, 6, 7 \\ 3 & \text{if } n = 5 \\ \lceil \frac{4n+5}{3} \rceil & \text{if } n \geq 8. \end{cases}$$

This completes the proof. □

3 Concluding remarks

The global equitable domination number for standard graphs like path, cycle, etc., is investigated by Basavanagoud and Teli [2], while we investigate this parameter for the degree splitting graphs obtained from path, shell, bistar, gear graph, helm, closed helm and flower graph.

References

[1] Basavanagoud, B., Patil, P. V. & Hosamani, S. M. (2012) Domination in degree splitting graphs, *Journal of Analysis and Computation*, 8 (1), 1–8.

[2] Basavanagoud, B. & Teli, V. V. (2015) Equitable global domination in graphs, *International Journal of Mathematical Archive*, 6 (3), 122–125.

[3] Gangadharappa, D. B. & Desai, A. R. (2011) On the dominating of a graph and its complement, *Journal of Mathematics and Computer Science*, 2 (2), 222–233.

[4] Haynes, T. W., Hedetniemi, S. T. & Slater, P. J. (1998) *Fundamentals of Domination in Graphs*, Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, New York.

[5] Haynes, T. W., Hedetniemi, S. T. & Slater, P. J. (1998) *Domination in Graphs – Advanced Topics*, Monographs and Textbooks in Pure and Applied Mathematics, Marcel Dekker, New York.

[6] Kulli, V. R. & Janakiram, B. (1996) The total global domination number of a graph, *Indian Journal of Pure and Applied Mathematics*, 27 (6), 537–542.

[7] Nellai Murugan, A. & Victor Emmanuel, G. (2013) Degree equitable domination number and independent domination number of a graph, *International Journal of Innovative Research in Science, Engineering and Technology*, 2 (11), 6419–6423.

[8] Ponraj, R. & Somasundaram, S. (2004) On the degree splitting graph of a graph, *Nat. Acad. Sci. Letters*, 27 (7–8), 275–278.

[9] Revathi, S. & Harinarayanan, C. V. R. (2014) Equitable domination in fuzzy graphs, *International Journal of Engineering Research and Applications*, 4 (6), 80–83.

[10] Sampathkumar, E. (1989) The global domination number of a graph, *Journal of Mathematical and Physical Sciences*, 23 (5), 377–385.

[11] Swaminathan, V. & Dharmalingam, K. (2011) Degree equitable domination on graphs, *Kragujevac Journal of Mathematics*, 35 (1), 191–197.

[12] Vaidya, S. K. & Pandit, R. M. (2013) Some new perspectives on global domination in graphs, *ISRN Combinatorics*, Vol. 2013, Article ID 201654, 4 pages.

[13] Vaidya, S. K. & Pandit, R. M. (2016) Global equitable domination number of some wheel related graphs, *International Journal of Mathematical Combinatorics*, 3, 77–85.

[14] Vaidya, S. K. & Pandit, R. M., Global equitable domination in Cartesian product of graphs, *Journal of Discrete Mathematical Sciences and Cryptography*, (communicated).

[15] West, D. B. (2003) *Introduction to Graph Theory*, Prentice-Hall of India, New Delhi.