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Structural sequences for primes using right-end-digits

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Abstract: Integers are expressed in the form nR where R represents the right-end-digits and n represents the digits to the left of R. n can be classified by the sequences $\{3t\}$, $\{3t + 1\}$, $\{3t + 2\}$. When $n = 3t + 2$, no primes with $R = 1$ or 7 can be formed with these n; when $n = 3t$ no primes can be formed with $R = 3$ or 9, but when $n = 3t + 1$, all REDs can form a prime within the constraints of imbedded sequences.

Keywords: Prime numbers, Composite numbers, Right-end-digits, Integer structure.
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1 Introduction

Integers are composed of a number of digits with a structure where the right-end-digits (REDs) are independent of the size of the integer, so that it is convenient to consider their form as nR where R is the RED and n contains the remaining digits to the left of R. For example, the integer 279856321 has R =1 and n = 27985632.

The n part belongs to the sequences $\{3t\}$, $\{3t + 1\}$, $\{3t + 2\}$ and imbedded sequences of the form $\{a + bj\}$ where $a < b$ and 3|b, with a and b constant for a given sequences, j a variable.

2 Imbedded sequences which block prime formation for particular REDs

These concepts are best illustrated by considering the REDs 1 and 7, and 3 and 9, as separate pairs since when $n \in \{3t\}$ with REDs of 3 or 9, these integers can never be prime, while REDs 1 and 7 with $n \in \{3t + 2\}$ can never form prime integers, but when $n \in \{3t + 1\}$, the integers could be primes. This can be more finely tuned with imbedded sequences which demonstrate selectivity for certain REDs, namely when n conforms to certain (a, b) pairs, the REDs will always be composite (Table 1).

To illustrate this structure n values for the two pairs of REDs 1,7 and 3,9 are listed in Tables 2 (a) and (b) and Table 3 for $n = 1$ to 100 with the imbedded sequences which prevent the formation of primes. Larger primes composed of 30 and 50 digits are shown in Tables 4 and 5.

RED	Main sequence	(a, b)
1	$3t$	(3,39), (3,93)[$n > 3$], (9,21), (9,39), (12,33), (36,57), (39,51), (84,87)
	$3t + 1$	(1,66)[$n > 1$], (16,21), (16,69), (22,39), (22,51), (34,66)
7	$3t$	(0,21), (18,33), (18,51), (24,39), (24,57), (66,69)
	$3t + 1$	(1,51)[$n > 1$], (7,21), (7,66), (37,39), (43,57)
1,7	$3t + 2$	No primes formed
3	$3t + 1$	(1,39), (13,21), (13,57), (25,33), (25,69), (49,51)
	$3t + 2$	(2,69)[$n > 2$), (14,33,(14,39), (20,21), (32,51), (32,57)
9	$3t + 1$	(1,57)[$n > 1$], (4,21), (16,39), (28,51), (31,33), (52,69), (64,231), (79,102)
	$3t + 2$	(11,21), (11,51), (20,33), (20,57), (29,39), (29,69), (32,42), (89,93)
3,9	$3t$	No primes formed

Table 1. Imbedded sequences blocking prime formation for particular REDs

n	***R***	**Type**	**Sequence**	
			***n* =**	***R*≠**
3	1	*p*		
	7	*p*		
6	1	*p*		
	7	*p*		
9	1	*c*	9 + 21*j*	1
	7	*p*		
12	1	*c*	12 + 33*j*	1
	7	*p*		
15	1	*p*		
	7	*p*		
18	1	*p*		
	7	*c*	18 + 33*j*	7
21	1	*p*		
	7	*c*	21*j*	7
24	1	*p*		
	7	*c*	24 + 57*j*	7
27	1	*p*		
	7	*p*		
30	1	*c*	9 + 21*j*	1
	7	*p*		
33	1	*p*		
	7	*p*		
36	1	*c*	36 + 57*j*	1
	7	*p*		
39	1	*c*	39 + 51*j*	1
	7	*p*		
42	1	*p*		
	7	*c*	21*j*	7
45	1	*c*	12 + 33*j*	1
	7	*p*		
48	1	*c*	9 + 39*j*	1
	7	*p*		
51	1	*c*	9 + 21*j*	1
	7	*c*	18 + 33*j*	7
54	1	*p*		
	7	*p*		
57	1	*p*		
	7	*p*		
60	1	*p*		
	7	*p*		
63	1	*p*		
	7	*c*	21*j*	7
66	1	*p*		
	7	*c*	66 + 69*j*	7
69	1	*p*		
	7	*c*	18 + 51*j*	7
72	1	*c*	9 + 21*j*	1
	7	*p*		
75	1	*p*		
	7	*p*		
78	1	*c*	12 + 33*j*	1
	7	*p*		
81	1	*p*		
	7	*c*	24 + 57*j*	7
84	1	*c*	84 + 87*j*	1
	7	*c*	18 + 33*j*	7
87	1	*c*	9 + 39*j*	1
	7	*p*		
90	1	*c*	39 + 51*j*	1
	7	*p*		
93	1	*c*	36 + 57*j* 9 + 21*j*	1
	7	*p*		
96	1	*c*	3 + 93*j*	1
	7	*p*		
99	1	*p*		
	7	*p*		
102	1	*p*		
	7	*c*	24 + 39*j*	7

p: prime

c: composite

Table 2 (a). $n = 3t$, $R \neq 1, 7$ means that no primes are formed with these R values and the n sequence $(a + bj)$

On the other hand Table 2b: $R \neq 1, 7$ indicates the imbedded sequences which prevent the n from ever forming primes with these n from $n = 1$ to $n = 100$ in steps of 3.

n	***R***	**Type**	**Sequence**		***n***	***R***	**Type**	**Sequence**		***n***	***R***	**Type**	**Sequence**	
			***n* =**	***R*≠**				***n* =**	***R*≠**				***N* =**	***R*≠**
1	1	*p*			34	1	*c*	$34 + 66j$	1	70	1	*p*		
	7	*p*				7	*p*				7	*c*	$7 + 21j$	7
4	1	*p*			37	1	*c*	$16 + 21j$	1	73	1	*c*	$22 + 51j$	1
	7	*p*				7	*c*	$37 + 39j$	7		7	*c*	$7 + 66j$	7
7	1	*p*			40	1	*p*			76	1	*p*		
	7	*c*	$7 + 21j$	7		7	*c*	$7 + 33j$	7		7	*c*	$37 + 39j$	7
10	1	*p*			43	1	*p*			79	1	*c*	$16 + 21j$	1
	7	*p*				7	*c*	$43 + 57j$	7		7	*p*		
13	1	*p*			46	1	*p*			82	1	*p*		
	7	*p*				7	*p*				7	*p*		
16	1	*c*	$16 + 21j$	1	49	1	*p*			85	1	*c*	$16 + 69j$	1
	7	*p*				7	*c*	$7 + 21j$	7		7	*p*		
19	1	*p*			52	1	*p*			88	1	*p*		
	7	*p*				7	*c*	$1 + 51j$	7		7	*p*		
22	1	*c*	$22 + 39j$	1	55	1	*c*	$55 + 57j$	1	91	1	*p*		
	7	*p*				7	*p*				7	*c*	$7 + 21j$	7
25	1	*p*			58	1	*c*	$16 + 21j$	1	94	1	*p*		
	7	*p*				7	*p*				7	*p*		
28	1	*p*			61	1	*c*	$22 + 39j$	1	97	1	*p*		
	7	*c*	$7 + 21j$	7		7	*p*				7	*p*		
31	1	*p*			64	1	*p*			100	1	*c*	$16 + 21j$	1
	7	*p*				7	*p*				7	*c*	$43 + 57j$	7
					67	1	*c*	$1 + 66j$	1					
						7	*p*							

p: prime

c: composite

Table 2 (b). $n = 3t + 1$, $R \neq 1, 7$ indicates that the imbedded sequences prevent the n from ever forming primes with these n

n	***R***	**Type**	**Sequence**		***n***	***R***	**Type**	**Sequence**		***n***	***R***	**Type**	**Sequence**	
			***n* =**	***R*≠**				***n* =**	***R*≠**				***n* =**	***R*≠**
1	3	*p*			34	3	*c*	$13 + 21j$	3	70	3	*c*	$13 + 57j$	3
	9	*p*				9	*p*				9	*p*		
4	3	*p*			37	3	*p*			73	3	*p*		
	9	*c*	$4 + 21j$	9		9	*p*				9	*p*		
7	3	*p*			40	3	*c*	$1 + 39j$	3	76	3	*c*	$13 + 21j$	3
	9	*p*				9	*p*				9	*p*		
10	3	*p*			43	3	*p*			79	3	*c*	$1 + 39j$	3
	9	*p*				9	*p*				9	*c*	$79 + 102j$	9
13	3	*c*	$13 + 21j$	3	46	3	*p*			82	3	*p*		
	9	*p*				9	*c*	$4 + 21j$	9		9	*p*		
16	3	*p*			49	3	*c*	$49 + 51j$	3	85	3	*p*		
	9	*c*	$16 + 39j$	9		9	*p*				9	*p*		
19	3	*p*			52	3	*p*			88	3	*p*		
	9	*p*				9	*c*	$52 + 69j$	9		9	*c*	$4 + 21j$	9
22	3	*p*			55	3	*c*	$13 + 21j$	3	91	3	*c*	$25 + 33j$	3
	9	*p*				9	*c*	$16 + 39j$	9		9	*p*		
25	3	*c*	$5 + 33j$	3	58	3	*c*	$25 + 33j$	3	94	3	*c*	$25 + 69j$	3
	9	*c*	$4 + 21j$	9		9	*c*	$1 + 57j$	9		9	*c*	$16 + 39j$	9
28	3	*p*			61	3	*p*			97	3	*c*	$13 + 21j$	3
	9	*c*	$28 + 51j$	9		9	*p*				9	*c*	$31 + 33j$	9
31	3	*p*			64	3	*p*			100	3	*c*	$49 + 51j$	3
	9	*c*	$31 + 33j$	9		9	*c*	$31 + 33j$	9		9	*p*		
					67	3	*p*							
						9	*c*	$4 + 21j$	9					

p: prime

c: composite

Table 3 (a). $n = 3t + 1$

n	*R*	Type	Sequence	
			n =	*R*≠
2	3	*p*		
	9	*p*		
5	3	*p*		
	9	*p*		
8	3	*p*		
	9	*p*		
11	3	*p*		
	9	*c*	$11 + 21j$	9
14	3	*c*	$14 + 33j$	3
	9	*p*		
17	3	*p*		
	9	*p*		
20	3	*c*	$20 + 21j$	3
	9	*c*	$20 + 33j$	9
23	3	*p*		
	9	*p*		
26	3	*p*		
	9	*p*		
29	3	*p*		
	9	*c*	$29 + 39j$	9
32	3	*c*	$32 + 51j$	3
	9	*c*	$11 + 21j$	9

n	*R*	Type	Sequence	
			n =	*R*≠
35	3	*p*		
	9	*p*		
38	3	*p*		
	9	*p*		
41	3	*c*	$20 + 21j$	3
	9	*p*		
44	3	*p*		
	9	*p*		
47	3	*c*	$14 + 33j$	3
	9	*p*		
50	3	*p*		
	9	*p*		
53	3	*c*	$14 + 39j$	3
	9	*c*	$11 + 21j$ $20 + 33j$	9
56	3	*p*		
	9	*p*		
59	3	*p*		
	9	*p*		
62	3	*c*	$20 + 21j$	3
	9	*c*	$11 + 51j$	9
65	3	*p*		
	9	*p*		
68	3	*p*		
	9	*c*	$29 + 39j$	9

n	*R*	Type	Sequence	
			n =	*R*≠
71	3	*c*	$2 + 69j$	3
	9	*p*		
74	3	*p*		
	9	*c*	$11 + 21j$	9
77	3	*p*		
	9	*c*	$20 + 57j$	9
80	3	*c*	$14 + 33j$	3
	9	*p*		
83	3	*c*	$20 + 21j$ $32 + 51j$	3
	9	*p*		
86	3	*p*		
	9	*c*	$20 + 33j$	9
89	3	*c*	$32 + 57j$	3
	9	*c*	$89 + 93j$	9
92	3	*c*	$14 + 39j$	3
	9	*p*		
95	3	*p*		
	9	*c*	$11 + 21j$	9
98	3	*p*		
	9	*c*	$29 + 69j$	9
101	3	*p*		
	9	*p*		

p: prime

c: composite

Table 3(b). $n = 3t + 2$

p	n	R	Remarks
6719980305 5971396836 1666935769	$3t + 1$ $10 + 21j$	9	This n could yield additional primes with $R = 1$, 3, or 7. $j = 3199990621712923658865080646$
5214196228 5665768942 3872613771	$3t + 1$ $13 + 21j$	1	This n could yield additional primes with $R = 7$ or 9, but $R = 3$ can never be prime with this n. $j = 24829505850317417568685107684$
1157569866 6830365787 8962467957	$3t$ $9 + 21j$	7	Main m sequence prevents $R = 3$ or 9 from forming primes with this n; the imbedded sequence has $R \neq 1$ so that $R = 7$ is the only R which forms a prime with this n. $j = 55122374603954122809029 7466$
5908726128 2517955133 6102196593	$3t + 2$ $11 + 21j$	3	Main m sequence prevents $R = 1$ or 7 from forming primes with this n; the imbedded sequence has $R \neq 9$ so that $R = 3$ is the only R which forms a prime with this n. $j = 28136791086913311968385 81888$

Table 4. Some 30-digit primes

p	n	R	Remarks
6449532778 1887693539 7385586910 6683910338 8567300449	$3t + 2$ $2 + 21j$	9	Main sequence prevents primes with $R = 1$ or 7. The imbedded sequence give a high yield for $R = 3$ or 9 which permits this n to form a prime with $R = 3$. $j = 3071206082470842549511359937669849481113741300 02$
3076254225 0301270692 0514605395 8616692729 1732754961	$3t$ $9 + 33j$	1	Main sequence prevents primes with $R = 3$ or 9. The imbedded sequence permits this to form primes with $R = 1$ or 7. $j = 932198250009129414910650319381398997796719174 39$
1545241701 1775787851 9510473095 6315938884 0946309807	$3t + 1$ $28 + 33j$	7	The two sequences allow this to have $R = 3$, 7 or 9 to form primes with this n. $j = 735829381513132754473859395693483780420 9$

Table 5. Some 50-digit primes

3 Concluding comments

It is clear that the various sequences occur because of the integer structure [2]. If we compare the modular rings, then the row sequences are sustained to infinity, and so the imbedded sequences should survive in the same way. This type of structural analysis is independent of the size of a domain.

References

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