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Combined 3-Fibonacci sequences from a new type

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Abstract: New combined 3-Fibonacci sequences are introduced and the explicit formulae for their n-th members are given.
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1 Introduction

The authors have introduced a series of extensions of the nature of the Fibonacci sequence in a series of papers and book [1]. One of these extensions [2] is the following: Let a, b, c, d, e, f be arbitrary real numbers. Let $\alpha_0 = a, \beta_0 = b, \gamma_0 = c, \alpha_1 = d, \beta_1 = e, \gamma_1 = f$, and for each natural number n:

$$\alpha_{n+2} = \sigma_{\alpha,1}\beta_{n+1} + \sigma_{\alpha,2}\gamma_{n+1} + \sigma_{\alpha,3}\beta_n + \sigma_{\alpha,4}\gamma_n,$$

$$\beta_{n+2} = \sigma_{\beta,1}\gamma_{n+1} + \sigma_{\beta,2}\alpha_{n+1} + \sigma_{\beta,3}\gamma_n + \sigma_{\beta,4}\alpha_n,$$

$$\gamma_{n+2} = \sigma_{\gamma,1}\alpha_{n+1} + \sigma_{\gamma,2}\beta_{n+1} + \sigma_{\gamma,3}\alpha_n + \sigma_{\gamma,4}\beta_n,$$

where $\sigma_{\alpha,1}, ..., \sigma_{\alpha,4}, \sigma_{\beta,1}, ..., \sigma_{\beta,4}, \sigma_{\gamma,1}, ..., \sigma_{\gamma,4} \in \{0, 1\}$ and

$$\sigma_{\alpha,1} + \sigma_{\alpha,2} = 1,$$

$$\sigma_{\alpha,3} + \sigma_{\alpha,4} = 1,$$

$$\sigma_{\beta,1} + \sigma_{\beta,2} = 1,$$

$$\sigma_{\beta,3} + \sigma_{\beta,4} = 1,$$

$$\sigma_{\gamma,1} + \sigma_{\gamma,2} = 1,$$

$$\sigma_{\gamma,3} + \sigma_{\gamma,4} = 1,$$

$$\sigma_{\alpha,1} + \sigma_{\alpha,3} = 1,$$

$$\sigma_{\alpha,2} + \sigma_{\alpha,4} = 1,$$

$$\sigma_{\beta,1} + \sigma_{\beta,3} = 1,$$

$$\sigma_{\beta,2} + \sigma_{\beta,4} = 1,$$

$$\sigma_{\gamma,1} + \sigma_{\gamma,3} = 1,$$

$$\sigma_{\gamma,2} + \sigma_{\gamma,4} = 1.$$

2 Main results

Here, a new type of combined 3-Fibonacci sequences are introduced.

Let again a, b, c, d, e, f be arbitrary real numbers, $\alpha_0 = a, \beta_0 = b, \gamma_0 = c, \alpha_1 = d, \beta_1 = e, \gamma_1 = f$, and for each natural number n:

$$\alpha_{n+2} = \beta_{n+1} + \gamma_n,$$

$$\beta_{n+2} = \beta_{n+1} + \beta_n,$$

$$\gamma_{n+2} = \beta_{n+1} + \alpha_n.$$

The first 10 values of sequences $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are the following

n	α_n	β_n	γ_n
0	a	b	c
1	d	e	f
2	$c+e$	$b+e$	$a+e$
3	$b+e+f$	$b+2e$	$b+d+e$
4	$a+b+3e$	$2b+3e$	$b+c+3e$
5	$3b+d+4e$	$3b+5e$	$3b+4e+f$
6	$4b+c+8e$	$5b+8e$	$a+4b+8e$
7	$8b+12e+f$	$8b+13e$	$8b+d+12e$
8	$a+12b+21e$	$13b+21e$	$12b+c+21e$
9	$21b+d+33e$	$21b+33e$	$21b+33e+f$
	$\ldots$	$\ldots$	$\ldots$

Let $\{F_n\}_{n=0}^{\infty}$ be the standard Fibonacci sequence, where $F_0 = 0, F_1 = 1, F_{n+2} + F_{n+1} + F_n$ for each natural number $n \geq 0$.

Theorem: For each natural number $n \geq 1$:

$$\alpha_{4n} = a + (F_{4n-1} - 1)b + F_{4n}e,$$

$$\beta_{4n} = F_{4n-1}b + F_{4n}e,$$

$$\gamma_{4n} = (F_{4n-1} - 1)b + c + F_{4n}e,$$

$$\alpha_{4n+1} = F_{4n}b + d + (F_{4n+1} - 1)e,$$

$$\beta_{4n+1} = F_{4n}b + F_{4n+1}e,$$

$$\gamma_{4n+1} = F_{4n}b + (F_{4n+1} - 1)e + f,$$

$$\alpha_{4n+2} = (F_{4n+1} - 1)b + c + F_{4n+2}e,$$

$$\beta_{4n+2} = F_{4n+1}b + F_{4n+2}e,$$

$$\gamma_{4n+2} = a + (F_{4n+1} - 1)b + F_{4n+2}e,$$

$$\alpha_{4n+3} = F_{4n+2}b + (F_{4n+3}e - 1) + f,$$

$$\beta_{4n+3} = F_{4n+2}b + F_{4n+3}e,$$

$$\gamma_{4n+3} = F_{4n+2}b + d + (F_{4n+3} - 1)e.$$

Proof: We can prove the Theorem, for example, by induction. For $n = 1$, the validity of the Theorem is checked directly from the above table. Let us assume that the Theorem is valid for some natural number $n \geq 1$. Then:

$$
\begin{aligned}
\alpha_{4n+4} &= \beta_{4n+3} + \gamma_{4n+2} \\
&= F_{4n+2}b + F_{4n+3}e + a + (F_{4n+1} - 1)b + F_{4n+2}e \\
&= a + (F_{4n+2} + F_{4n+1} - 1)b + (F_{4n+3} + F_{4n+2})e \\
&= a + (F_{4n+3} - 1)b + F_{4n+4}e \\
&= a + (F_{4(n+1)-1} - 1)b + F_{4(n+1)}e.
\end{aligned}
$$

The rest formulas are checked by analogy. □

3 Conclusion

Two new schemes, modifying standard form of 2- and 3- Fibonacci sequences, will be discussed in near future.

References

[1] Atanassov K., Atanassova, V., Shannon, A., Turner, J. (2002) *New Visual Perspectives on Fibonacci Numbers*. World Scientific, New Jersey.

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