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Fibonacci numbers with prime subscripts: Digital sums for primes versus composites

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Abstract: If we use the expression $F_p = kp \pm 1$, p prime, then digital sums of k reveal specific values for primes versus composites in the range $7 \le p \le 107$. The associated digital sums of $F_{p\pm1}$ also yield prime/composite specificity. It is shown too that the first digit of F_p, and hence for the corresponding triples, $(F_p, F_{p\pm1})$ and (F_p, F_{p-1}, F_{p-2}) can be significant for primality checks.

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1 Introduction

The structure of a recursive sequence such as the Fibonacci series is, by definition, very regular [2, 3], so that any fluctuations can be analysed to distinguish between primes and composites when the subscripts or the order in the set are prime numbers [4, 5, 6]. We have recently shown [6] that Fibonacci numbers with prime subscripts have factors of the form $kp \pm 1$ (k even). Here we continue this analysis.

2 $F_p = kp \pm 1$: Digit sums of k

Since $F_p = F_p \times 1$, this function applies generally, as was shown for primitive Fibonacci triples [4] (Table 1). The k values have digit sums [6] which, like those for other F_p functions can often distinguish between primes and composites (Table 2). The right-end-digit (RED) for k, designated by k^*, has distinct values for a given p^*, irrespective of primality (Table 3).

p	F_p	*k*	Sign‡	Type
7	13	2	–	*p*
11	89	8	+	*p*
13	233	18	–	*p*
17	1597	94	–	*p*
19	4181	220	+	*c*
23	28657	1246	–	*p*
29	514229	17732	+	*p*
31	1346269	43428	+	*c*
37	24157817	652914	–	*c*
41	165580141	4038540	+	*c*
43	433494437	10081266	–	*p*
47	2971215073	63217342	–	*p*
53	53316291173	100596778	–	*c*
59	956722026041	1621562750	+	*c*
61	2504730781961	41061160360	+	*p*
67	44945570212853	670829406162	–	*c*
71	308061521170129	4338894664368	+	*p*
73	806515533049393	11048157986978	–	*p*
79	14472334024676221	183194101578180	+	*c*
83	99194853094755497	1195118711985006	–	*p*
89	1779979416004714189	19999768719154092	+	*c*
97	83621143489848422977	862073644225241474	–	*p*
101	573147844013817084101	5674731128849674100	+	*p*
103	1500520536206896083277	14568160545698020226	–	*p*
107	10284720757613717413913	96118885585174929102	–	*c*

Table 1. Digit sums of *k* (*p*: prime; *c*: composite)
‡ ['+' ≡ $p^* \in \{1, 9\}$; '–' ≡ $p^* \in \{3, 7\}$]

*p**	Primes	Composites
1	1, 2, 8, 9	3, 6
3	2, 4, 6, 8, 9	7
7	1, 2, 4, 8	6, 9
9	2	3, 4, 6, 8

Table 2. Digit sums of *k*

*p**	*k**
1	0+, 8+
3	6–, 8–
7	2–, 4–
9	0+, 2+

Table 3. REDs

3 First and last digits of F_p

The distribution of these is displayed in Table 4. A comparison of primes with composites (Table 5) illustrates that no distinction exists for the last digit. However, the first digit displays distinctions except when p = 7 with 2 as a common digit.

The first digit of each Fibonacci number occurs at a specific position, *n*, in the series. The following positions occur in a regular pattern as shown by ($n_j - n_{j-1}$) in Table 6. As noted above, the first digit of F_p seems to offer a distinction between primes and composites

1st digit → / Last ↓	1	2	3	4	5	6	7	8	9
1	✓	✓	✓	✓	✓	✓	✓	✗	✓
2	✓	✓	✓	✓	✓	✓	✗	✓	✗
3	✓	✓	✗	✓	✓	✓	✓	✓	✗
4	✓	✓	✓	✓	✗	✗	✓	✓	✗
5	✓	✓	✓	✗	✓	✓	✓	✗	✓
6	✓	✓	✓	✓	✗	✓	✗	✗	✓
7	✓	✓	✓	✓	✓	✗	✗	✗	✓
8	✓	✓	✓	✓	✗	✓	✗	✗	✗
9	✓	✓	✓	✓	✓	✗	✓	✓	✗
0	✓	✓	✓	✗	✓	✓	✗	✓	✗

Table 4. First and last digits of F_p

p*	**Primes**		**Composites**	
	1st digit	**Last digit**	**1st digit**	**Last digit**
1	2, 8, 3	1, 9	1	1, 9
3	2, 4, 8, 9	3, 7	5	3
7	1, 2	3, 7, 3	2, 4	3, 7
9	5	9	1, 4, 9	1, 9

Table 5. Comparison of primes and composites

1st digit	**1st n_j**	**($n_j - n_{j-1}$) patterns**
1	7	5, 5, 4, 1, 4, 1, 4, 5, 4, 1, 4, 1, 4, 5, 5, 4, 1, 4, 1, 4, 5, …
2	8	5, 5, 5, 9, 5, 5, 5, 5, 5, 9, 5, 5, 5, 9, …
3	4	5, 5, 1, 4, 5, 5, 1, 4, 5, 1, 4, 5, 5, 1, 4, 5, 5, …
4	19	5, 1, 9, 5, 14, 5, 5, 14, 5, 19, 5, …
5	5	5, 19, 5, 19, 5, 19, 19, 5, …
6	15	5, 19, 24, 19, 5, 19, …
7	25	19, 5, 19, 24, 19, …
8	6	5, 19, 24, 19, 5, …
9	16	43, 24, 19, …

Table 6. Some patterns

Since only $n = p$ yields primes these are sieved out (Table 7) [1] and they exhibit regular patterns.

N_1	$n = p$ has 1st digit N_1		$p_i - p_{i-1}$
	Primes	Composites	
1	7, 17	31, 41, 79	10, 14, 10, 38
2	13, 23, 47, 61	37	10, 14, 10, 14
3	71	-	-
4	43	19, 43	24, 24
5	5, 29, 101	53	24, 24, 48
6	-	-	-
7	-	-	-
8	11, 73, 97	-	62, 24
9	83	-	-

Table 7. First digit parities

In Table 7 for first digits equal to 3, 8 or 9, the corresponding F_p are all primes. There are no prime values when the first digit is 6 or 7, but a first digit of 1 or 2 gives the most values of p with a mixture of primes and composites.

4. F_p neighbours

To calculate primitive Fibonacci triples [4] the relationships set out in Table 8 were used

p^*	F_{p-1}	F_{p+1}
1	Kp	$Kp \pm 1$
3	$Kp \pm 1$	Kp
7	$Kp \pm 1$	Kp
9	Kp	$Kp \pm 1$

Table 8

p^*	Digit sum of K	
	Primes	Composites
1	1, 5, 9	2, 3
3	1, 2, 3, 6, 9	1, 9
7	3, 8, 9	6, 8, 9
9	6	2, 3, 4, 8

Table 9

The values of K were calculated ($F_{p\pm1} / p$) in the range $7 \le p \le 107$ and the digital sum of K were compared for primes and composites (Table 9). The distributions are clear for $p^* = 1, 9$, but $p^* = 3, 7$ have overlaps.

A better result is obtained if we compare the digit sum of individual components of Fibonacci number triples ($F_{p-2} + F_{p-1} = F_p$) in the range $7 \le p \le 107$ (24 primes). The results in Table 10 show parity distinction, which provide a further guide to primality.

p*	Digit sums of F_{p-2}		Digit sums of F_{p-1}		Digit sums of F_p	
	Primes	Composites	Primes	Composites	Primes	Composites
1	2, 7, 8	5, 7	1, 3, 8, 9	6, 8	1, 8	4
3	1, 2, 5, 7, 8	2	1, 8, 9	3	1, 8	4, 5
7	1, 2, 5, 7	4, 8	6, 8, 9	1, 9	1, 4	5, 8
9	2	4, 5, 7	3	1, 6, 8	5	4, 5, 8

Table 10. Fibonacci number triple digit sum parities

5 Concluding comments

Further analysis along these lines can be made so that indications of primality build up and increase the probability of testing the primality of F_p. The results outlined here can then be extended to consider probabilistic primality testing [7].

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