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Remark on twin primes

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Abstract: Recently, I. Gueye [2] proved a variant of Clement's Theorem on twin primes. We show that, this result follows by a simple identity.
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1 Introduction

Clement's Theorem on twin primes (see [1, 2, 3]) states that $n+2$ and $n+4$ are both primes if and only if $4.[(n+1)!+1]+n+2$ is divisible by $(n+2)(n+4)$.

Recently, I. Gueye [2] has proved the following variant: $n+2$ and $n+4$ are a couple of primes if and only if $n(n+1)!-2$ is divisible by $(n+2)(n+4)$.

The aim of this note is to offer a simple proof of this result.

2 The proof

Remark that the following identity holds true:

$$n.[4(n+1)!+n+6]=4.[n(n+1)!-2]+(n+2)(n+4) \tag{1}$$

Assume first that $(n+2)(n+4)$ divides $4(n+1)!+n+6$. Then, by Clement's Theorem, $n+2$ and $n+4$ are primes. By Identity (1), $n+2$ and $n+4$ divide $n(n+1)!-2$.

Reciprocally, suppose that $(n+2)(n+4)$ divides $n(n+1)!-2$.

As $n+2$ divides $n(n+1)!-2$, n must be odd. Indeed, otherwise $(n+2)/2$ would be an integer, which divides $n(n+1)!-1$.

On the other hand, as $(n+2)/2<n+1$, $(n+2)/2$ divides $(n+1)!$, which is a contradiction.

Since n is odd, one has $(n, (n+2)(n+4))=1$, so $(n+2)(n+4)$ divides $4(n+1)!+n+6$. Thus Clement's Theorem may be applied. □

References

[1] Clement, P. A. Congruences for sets of primes, *Amer. Math. Monthly*, Vol. 56, 1949, 23–25.

[2] Gueye, I. A note on twin primes, *South Asian J. Math*., Vol. 2, 2012, No. 2, 159–161.

[3] Ribenboim, P. *The New Book of Prime Number Records*, 3rd ed., Springer Verlag, 1996.