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Pulsated Fibonacci sequence. Part 2

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Abstract: Second type of Pulsated Fibonacci sequence is introduced and explicit formulas for the form of its members are formulated and proved.
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On 29 Sept. 2013, I wrote the first part of the paper and on the next day I sent it to my colleague and friend Prof. Tony Shannon for his 75th birtday and included it in previous number of the journal (see [1]), that was dedicated to him. Soon after this, I saw that the introduced sequence can be further modified and extended. In the present paper, such a modification of the Pulsated Fibonacci sequence is given and in a next research another extension of the new type of Fibonacci sequences will be described.

In [1] it was mentioned that during the last century, a lot of extensions and modifications of the Fibonacci sequence were introduced. Shannon and I defined some of them (see, e.g., our book [2]).

In [1], continuing this direction of research related to Fibonacci sequences, a new type of Fibonacci like sequence was introduced, as follows.

Let a and b be two fixed real numbers. Let us construct the following two sequences

$$\alpha_0 = a, \ \beta_0 = b$$

$$\alpha_{2k+1} = \beta_{2k+1} = \alpha_{2k} + \beta_{2k},$$

$$\alpha_{2k+2} = \alpha_{2k+1} + \beta_{2k},$$

$$\beta_{2k+2} = \beta_{2k+1} + \alpha_{2k},$$

for the natural number $k \geq 0$. This pair of sequences we called in [1] *Pulsated Fibonacci sequence.* Now, let us call it *(a, b)-Pulsated Fibonacci sequence.*

Now, we introduce a modification of the above sequence.

Let a, b and c be three fixed real numbers. Let us construct the following two sequences

$$\alpha_0 = a,\ \beta_0 = b$$

$$\alpha_1 = \beta_1 = c$$

$$\alpha_{2k} = \alpha_{2k-1} + \beta_{2k-2},$$

$$\beta_{2k} = \beta_{2k-1} + \alpha_{2k-2},$$

$$\alpha_{2k+1} = \beta_{2k+1} = \alpha_{2k} + \beta_{2k},$$

for the natural number $k \geq 1$. This pair of sequences we call *(a,b;c)-Pulsated Fibonacci sequence.*

The first values of the new sequence are given in the following

TABLE

n	α_n	$\alpha_n = \beta_n$	β_n
0	a	$-$	b
1	$-$	c	$-$
2	$b+c$	$-$	$a+c$
3	$-$	$a+b+2c$	$-$
4	$2a+b+3c$	$-$	$a+2b+3b$
5	$-$	$3a+3b+6c$	$-$
6	$4a+5b+9c$	$-$	$5a+4b+9c$
7	$-$	$9a+9b+18c$	$-$
8	$14a+13b+27c$	$-$	$13a+14b+27c$
9	$-$	$27a+27b+54c$	$-$
10	$40a+41b+81c$	$-$	$41a+40b+81c$
11	$-$	$81a+81b+162c$	$-$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Theorem. For every natural number $k \geq 0$,

$$\alpha_{2k+1} = \beta_{2k+1} = 3^{k-1}a + 3^{k-1}b + 2.3^k c, \tag{1}$$

$$\alpha_{4k+2} = \frac{3^{2k}-1}{2}a + \frac{3^{2k}+1}{2}b + 3^{2k}c, \tag{2}$$

$$\beta_{4k+2} = \frac{3^{2k}+1}{2}a + \frac{3^{2k}-1}{2}b + 3^{2k}c. \tag{3}$$

$$\alpha_{4k} = \frac{3^{2k-1}+1}{2}a + \frac{3^{2k-1}-1}{2}b + 3^{2k-1}c,\ \text{for } k \geq 1, \tag{4}$$

$$\beta_{4k} = \frac{3^{2k-1}-1}{2}a + \frac{3^{2k-1}+1}{2}b + 3^{2k-1}c,\ \text{for } k \geq 1. \tag{5}$$

Proof. Obviously, for $k = 0$ the assertion is valid. Let us assume that for some natural number

$k > 0$, (1)-(5) are valid. For the natural number $k+1$, first, we check that

$$\alpha_{4k+1} = \beta_{4k+1} = \alpha_{4k} + \beta_{4k} = \frac{3^{2k-1}+1}{2}a + \frac{3^{2k-1}-1}{2}b + 3^{2k-1}c + \frac{3^{2k-1}-1}{2}a + \frac{3^{2k-1}+1}{2}b + 3^{2k-1}c = 3^{2k-1}a + 3^{2k-1}b + 2.3^{2k-1}c.$$

Second, we check that

$$\alpha_{4k+2} = \alpha_{4k+1} + \beta_{4k} = 3^{2k-1}a + 3^{2k-1}b + 2.3^{2k-1}c + \frac{3^{2k-1}-1}{2}a + \frac{3^{2k-1}+1}{2}b + 3^{2k-1}c = \frac{3^{2k}-1}{2}a + \frac{3^{2k}+1}{2}b + 3^{2k}c.$$

All other equalities are checked analogously. □

For example, if $b = -a$, then the Pulsated Fibonacci sequence has the form:

n	α_n	$\alpha_n = \beta_n$	β_n
0	a	$-$	$-a$
1	$-$	c	$-$
2	$-a+c$	$-$	$a+c$
3	$-$	$2c$	$-$
4	$a+3c$	$-$	$-a+3b$
5	$-$	$6c$	$-$
6	$-a+9c$	$-$	$a+9c$
7	$-$	$18c$	$-$
8	$a+27c$	$-$	$-a+27c$
9	$-$	$54c$	$-$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

while, if $b = a$, then the Pulsated Fibonacci sequence has the form:

n	α_n	$\alpha_n = \beta_n$	β_n
0	a	$-$	a
1	$-$	$a+c$	$-$
2	$2a+c$	$-$	$2a+c$
3	$-$	$2a+2c$	$-$
4	$4a+3c$	$-$	$4a+3b$
5	$-$	$8a+6c$	$-$
6	$12a+9c$	$-$	$12a+9c$
7	$-$	$24a+18c$	$-$
8	$36a+27c$	$-$	$36a+27c$
9	$-$	$72a+54c$	$-$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The two sequences, discussed in [1] and here, can be called *2-Pulsated Fibonacci sequences* (from (a, b) and $(a, b; c)$-types). In a next paper, we will discuss the case with s-Pulsated Fibonacci sequences, where $s \geq 3$.

References

[1] Atanassov, K., Pulsating Fibonacci sequences. *Notes on Number Theory and Discrete Mathematics*, Vol. 19, 2013, No. 3, 12–14.

[2] Atanassov K., V. Atanassova, A. Shannon, J. Turner, *New Visual Perspectives on Fibonacci Numbers*. World Scientific, New Jersey, 2002.