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A note on switching in symmetric n-sigraphs

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Abstract: In this note, we define switching in a different manner and obtained some results on symmetric n-sigraphs.
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1 Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [2]. We consider only finite, simple graphs free from self-loops.

Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ ($\mu : V \to H_n$) is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/ symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(S_n)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [7], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P. Siva Kota Reddy [4]):

Definition. Let $S_n = (G, \sigma)$ be an n-sigraph. Then,
(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and
(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.
Note: An i-balanced n-sigraph need not be balanced and conversely.
The following characterization of i-balanced n-sigraphs is obtained in [7].

Proposition 1. **(E. Sampathkumar et al. [7])** *An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.*

In [7], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [5, 6] & [9]-[12]).

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

Proposition 2. **(E. Sampathkumar et al. [7])** *Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.*

2 New version of switching

Let $S_n = (G, \sigma)$ be an n-sigraph and $v \in V(S)$. The n-sigraph $S_n^v = (G, \sigma^v)$ is said to be obtained from S_n by *switching* v. The n-tuple $\sigma(A)$ is

$$\sigma^v(e) = \begin{cases} -\sigma(e), & \text{if } v \text{ is an end point of } e \\ \sigma(e), & \text{otherwise.} \end{cases}$$

Proposition 3. *Let $S_n = (G, \sigma)$ be an n-sigraph and let u and v be two vertices of S_n. Then $(S_n^u)^v = (S_n^v)^u$.*

Proof. If $(\sigma^v)^u = (\sigma^u)^v$ then proof is over. There are three cases: e has neither u or v as a vertex, e has exactly one of u or v as a vertex, or e has both u and v as vertices. If e is incident to neither u nor v then $(\sigma^v)^u(e) = \sigma^v(e) = \sigma(e) = \sigma^u(e) = (\sigma^u)^v(e)$. If e is incident to only one of u or v, then without loss of generality we let e be incident to v. Now we see that $(\sigma^v)^u(e) = \sigma^v(e) = -\sigma(e) = -\sigma^u(e) = (\sigma^u)^v(e)$. Finally, if e is incident to both u and v, then $(\sigma^v)^u(e) = -\sigma^v(e) = \sigma(e) = -\sigma^u(e) = (\sigma^u)^v(e)$. □

For $U \subseteq V(S_n)$, S_n^U is the n-sigraph obtained by switching each of the vertices of U. By Proposition 3, the order in which the vertices are switched does not matter. An n-sigraph $S'_n = (G', \sigma')$ is switching equivalent to $S_n = (G, \sigma)$, if $S'_n = (G', \sigma') \cong S_n^U$ for some $U \subseteq V(S_n)$. The set of n-sigraphs switching equivalent to $S_n = (G, \sigma)$ is called the switching class of $S_n = (G, \sigma)$, written $[S_n]$.

For any $a \in \{+, -\}$, let $\overline{a} \in \{+, -\} \setminus \{a\}$. In an n-tuple $(a_1, a_2, ..., a_n)$, the elements $a_{\lceil \frac{n}{2} \rceil}$ and $a_{\lceil \frac{n+1}{2} \rceil}$ are called *middle elements*. Note that an n-tuple has two middle elements if n is even and exactly one if n is odd. We now define various operations on an n-tuple $(a_1, a_2, ..., a_n)$ as follows:

i) *f-complement* , $(a_1, a_2, ..., a_n)^f = (\overline{a}_1, \overline{a}_2, ..., \overline{a}_n)$

ii) *m-complement* $(a_1, a_2, ..., a_n)^m = (b_1, b_2, ..., b_n)$ where ,

$$b_k = \begin{cases} \overline{a}_k, & \text{if } a_k \text{ is a middle element;} \\ a_k, & \text{Otherwise} \end{cases}$$

iii) *e- complement* $(a_1, a_2, ..., a_n)^e = (b_1, b_2, ..., b_n)$ where ,

$$b_k = \begin{cases} \overline{a}_k, & \text{if } a_k \text{ is not a middle element;} \\ a_k, & \text{Otherwise} \end{cases}$$

Let $t \in \{f, e, m\}$. Then *t-complement* S_n^t of an n-sigraph $S_n = (G, \sigma)$ is obtained from S_n by replacing each n-tuple on the edges of S_n by its t-complement.

Proposition 4. *Let $U \subseteq V(S_n)$. The graph obtained by f-complement of n-tuples on the edges in the cut $[U; U^f]$ is S_n^U.*

Proof. The only way for an edge of S_n^U to have a different n-tuple $(a_1, a_2, ..., a_n)$ than it had in S_n is if it has exactly one endpoint in U. If it has none, its n-tuple $(a_1, a_2, ..., a_n)$ never changes. On the other hand if it has two endpoints in U then it is f-complement twice (once for each endpoint in U). Thus the two n-sigraphs are the same. □

Proposition 5. *If $S_n = (G, \sigma)$ is i-balanced then so is any switching of S_n.*

Proof. Let $U \subseteq V(S_n)$. By Proposition 4, S_n^U is obtained by f-complement of n-tuples on the edges in the cut $[U; U^f]$. The intersection of a cycle with a cut must always contain the number of n-tuples in any cycle whose k^{th} co-ordinate is $-$ is even, and therefore f-complement those edges has no change on their product(it is again i-balanced). Thus the n-tuple of a cycle remains unchanged. □

Proposition 6. *The switching class $[S_n]$ contains only identity n-tuples if, and only if, $S_n = (G, \sigma)$ is i-balanced.*

Remark. In [1], the author introduced above switching for signed graphs. In this note, we generalized this switching for n-sigraphs. Further, the above new type switching defined in n-sigraphs is coincidence with switching already defined in n-sigraphs for there exists an n-marking of n-sigraphs.

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