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On construction of rhomtrees as graphical representation of rhotrices

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Abstract: We introduce the concept of rhomtrees as a graphical method of representing rhotrices and present the relationships of their graphs with existing graphical models of some real world situations. These models include the topology of computing network, energy resource distribution network, methane compound and certain products of sets.
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1. Introduction

We adopt the concept of trees in graph theory, to present our graphical imagination for representing rhotrices of dimension *n*, termed in this paper as rhomtrees of order *m*, such that $n \in 2\mathrm{Z}^+ + 1$ and $m = \frac{1}{2}(n^2+1)$.

Rhotrix theory is a relatively new paradigm of matrix theory, whose goal is central on representation of arrays of numbers in rhomboid mathematical form. A set of all rhotrices of dimension *n*, denoted as $\hat{R}(n)$, was defined in the work of Mohammed [7] as

$$\hat{R}(n)=\left\{\left\langle\begin{array}{ccccccccccc} &&&&& r_1 &&&&& \\ &&&& r_2 & r_3 & r_4 &&&& \\ && \vdots & \vdots && \vdots && \vdots & \vdots && \\ & \vdots & \vdots & \vdots && \vdots && \vdots & \vdots & \vdots & \\ r_{\left\{\frac{\frac{1}{2}(n^2+1)+1}{2}\right\}-n\backslash 2} & \cdots & \cdots & \cdots && r_{\left\{\frac{\frac{1}{2}(n^2+1)+1}{2}\right\}} && \cdots & \cdots & \cdots & r_{\left\{\frac{\frac{1}{2}(n^2+1)+1}{2}\right\}+n\backslash 2} \\ & \vdots & \vdots & \vdots && \vdots && \vdots & \vdots & \vdots & \\ && \vdots & \vdots && \vdots && \vdots & \vdots && \\ &&&& r_{\frac{1}{2}(n^2-5)} & r_{\frac{1}{2}(n^2-3)} & r_{\frac{1}{2}(n^2-1)} &&&& \\ &&&&& r_{\frac{1}{2}(n^2+1)} &&&&& \end{array}\right\rangle : r_1,\ldots,r_{\frac{1}{2}(n^2+1)} \in \Re \ and\ n \in 2Z^+ + 1\right\}$$

where $h(R) = r_{\left\{\frac{\frac{1}{2}(n^2+1)+1}{2}\right\}}$ is called the heart of any rhotrix $R(n) \in \hat{R}(n)$. The concept of rhotrices commenced from the work of Ajibade [1], when he introduced the initial algebra and analysis of three dimensional rhotrices, as an extension of ideas on matrix-tertions and matrix-noitrets suggested by Atanassov and Shannon [2]. Ever since then, there has been much interest by researchers to improve upon the development of rhotrix theory. For the review of developments in the literature of rhotrix theory, one can see [1], [3], [4], [5], [6], [7], [8] and [9].

Our interest to propose the graphical model for representing rhotrices was aroused by the name 'heart' , denoted as h(R), given to the element at a particular intersection of the two leading diagonal entries of any given rhotrix R, and the significant role the hearts of rhotrices play during multiplication of rhotrices of the same dimension. For example, let

$$R=\left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle, \quad Q=\left\langle \begin{array}{ccc} & f & \\ g & h(Q) & j \\ & k & \end{array} \right\rangle$$

be any three dimensional rhotrices then their product is,

$$R\circ Q=\left\langle \begin{array}{ccc} & ah(Q)+fh(R) & \\ bh(Q)+gh(R) & h(R)h(Q) & dh(Q)+jh(R) \\ & eh(Q)+kh(R) & \end{array} \right\rangle.$$

This multiplication was thereafter extended to rhotrices of dimension n in form of generalization by Mohammed [7].

In this paper, we introduce the concept of constructing a rhomtree of order $m=\frac{1}{2}(n^2+1)$ and present it as a graphical representation of rhotrix of dimension n, where $n\in 2Z^{+}+1$. The developed aforesaid tool can serve as a pictorial form of presenting realistic connection and interrelationship between the 'heart entry' and the 'non heart entries' of any given rhotrix of dimension n. Also we uncover some relationships of these rhomtrees with existing graphical models of some real world situations such as the topology of computing network, energy resource distribution network, methane compound and certain products of sets.

1.1. Definition (Rhomtree)

A rhomtree is a graphical tree $T(m)$ of order $m=\frac{1}{2}(n^2+1)$ such that $n\in 2Z^{+}+1$, whose root is incident with four vertices or four components of binary branches. As examples, we can have the following fig. 1(a), (b) and (c) representing rhomtrees of order 5, 13 and 25 respectively:

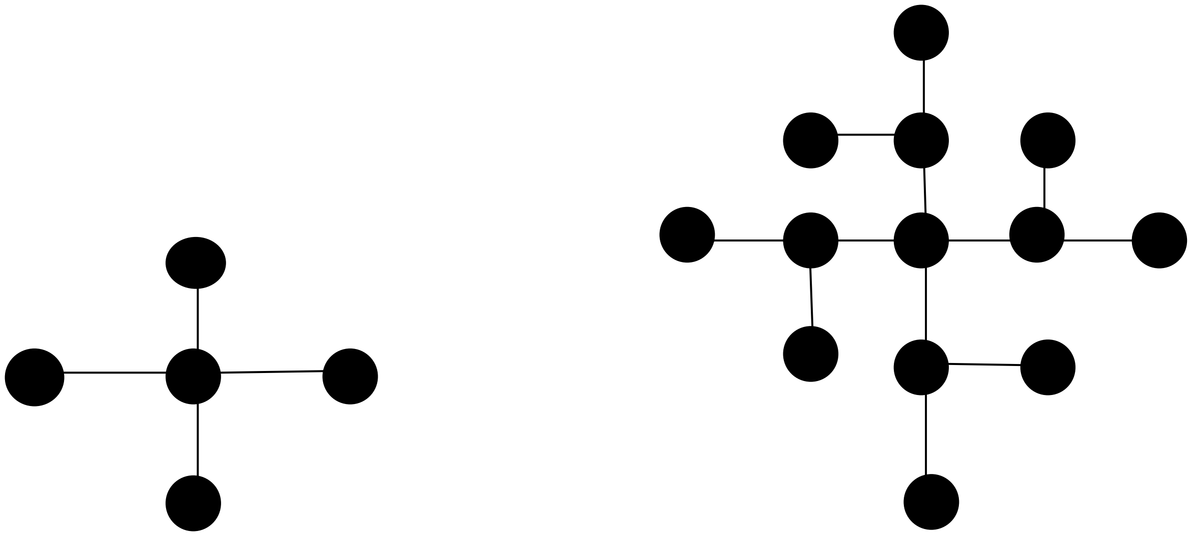
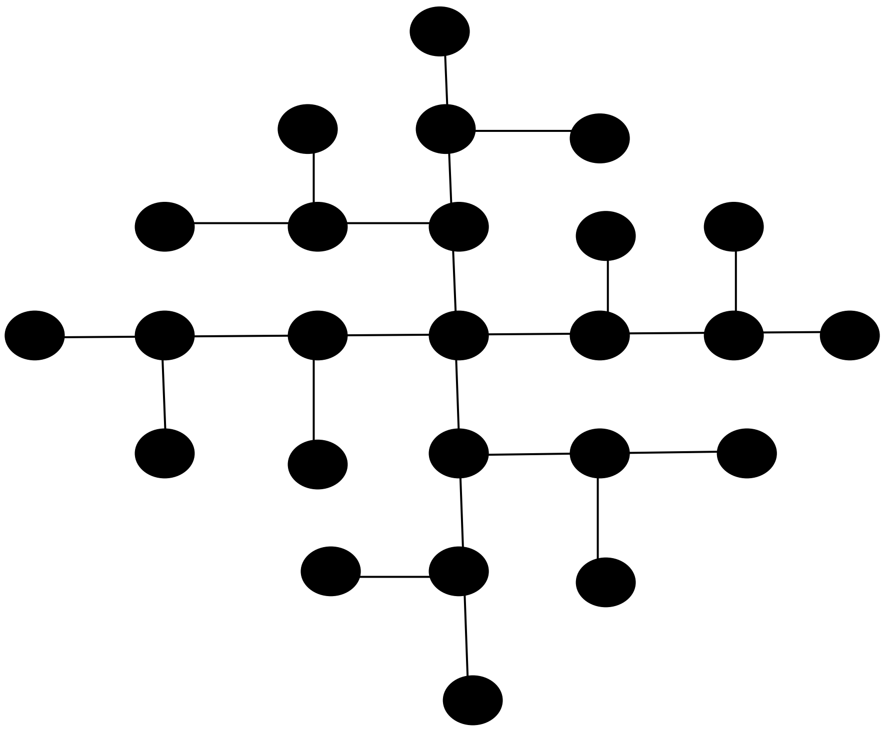


Fig. 1 **(a)** Rhomtree of order 5 **(b)** Rhomtree of order 13



(c) Rhomtree of order 25

2. Graphical representation of a rhotrix as a rhomtree

To achieve our imagination of representing real rhotrix of dimension n as a graphical tree into practice, which we named rhomtree, the algebra and analysis of trees in the theory of graph (see Seymour and Marc Lars [10]) will be adopted in the construction, in such a way that, all the entries in rhotrix that were originally real numbers are now considered to be the labels of node points of the graph of a rhomtree. These labeled vertices are connected by directed or undirected edges to form a tree, whose root is incident with four vertices and these four vertices may branched into binary components of other vertices. This rhomtree is of order $m = \frac{1}{2}(n^2 + 1)$, since it graphically represents rhotrix of dimension *n*, such that $n \in 2Z^+ + 1$. Thus, rhomtrees representation of rhotrices of dimension *n*, using a particular scenario for *n* will be discussed in the following subsections and their comparison with some existing modeling of real world situations will be given in section 3.

2.1 Graphical representation of rhotrix *R(3)*

Let $\hat{R}(3)$ be a set consisting of all real rhotrices of dimension three and let *R(3)* be any element in $\hat{R}(3)$ given by

$$R(3) = \left\langle \begin{matrix} & r_1 & \\ r_2 & r_3 & r_4 \\ & r_5 & \end{matrix} \right\rangle$$

Clearly, if we take each entry in $R(3)$ as a node point and then connect all of the entries as network of five vertices using a particular pattern or style for the construction, in such a way that the heart vertex will serve as the root of the tree, while the non heart vertices will serve as branches, then we obtain a rhomtree *T(5)* corresponding to the rhotrix $R(3)$ as shown in Fig. 2.

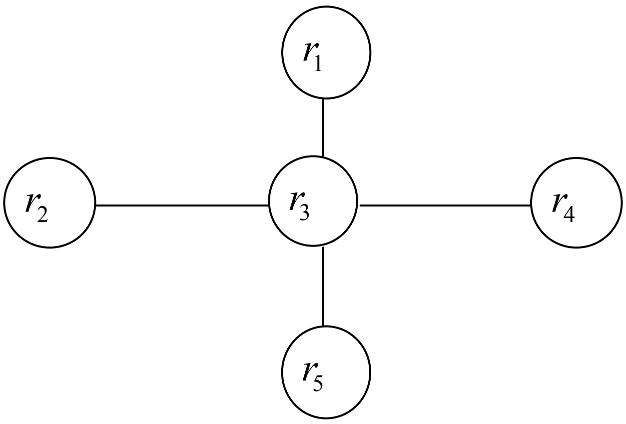


Fig. 2: Rhomtree *T(5)* representing any rhotrix in $\hat{R}(3)$.

This rhomtree *T(5)* has a root labeled as $h(R) = r_3$ which branches into four connected components of vertices r_1, r_2, r_4 and r_5.

2.2 Graphical modeling of rhotrix $\hat{R}(5)$

Let $\hat{R}(5)$ be a set consisting all real rhotrices of dimension five and let *R(5* be any element in $\hat{R}(5)$ given by

$$R(5) = \left\langle \begin{array}{ccccc} & & r_1 & & \\ & r_2 & r_3 & r_4 & \\ r_5 & r_6 & r_7 & r_8 & r_9 \\ & r_{10} & r_{11} & r_{12} & \\ & & r_{13} & & \end{array} \right\rangle$$

where $h(R) = r_7$ is the heart of $R(5)$.

Now, we can construct the graph of rhomtree *T(13)* by extending the graph of rhomtree *T(5)*. That is, if we take each entry in $R(5)$ as a node point and then connect all the entries as network of thirteen vertices using a particular pattern or style for the construction, in such a way that the heart vertex is adjacent with four vertices while the non heart vertices are binary branches of four components connected to the root vertex then we obtain a rhomtree *T(13)* corresponding to the rhotrix $R(5)$ as shown in Fig. 3.

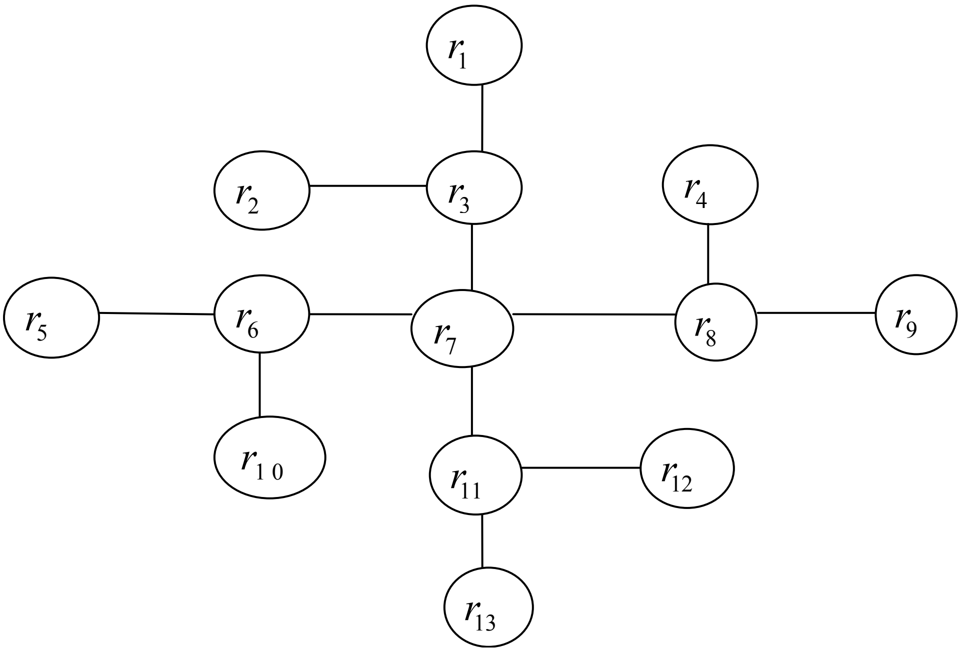


Fig. 3: Rhomtree *T(13)* representing any rhotrix in $\hat{R}(5)$

Note that this rhomtree *T(13)* has a root labeled as $h(R) = r_7$ that bridged four connected components of binary branches.

2.3 Graphical modeling of rhotrix $\hat{R}(n)$

Let $\hat{R}(n)$ be a set consisting all real rhotrices of dimension $n \in 2Z^+ + 1$ and let *R(n)* be any rhotrix in $\hat{R}(n)$ then the graphical representation of rhotrix *R(n)* is a rhomtree *T(m)* consisting of $m = \frac{1}{2}(n^2 + 1)$ number of vertices and $\frac{1}{2}(n^2 - 1)$ number of edges, having four components of binary branches and each component is bridged to the root vertex by one incident edge.

Remark 2.3.1

There exists a homeomorphism between any two rhomtrees of order $m = \frac{1}{2}(n^2 + 1)$, $n \in 2Z^+ + 1$. This is clear since rhomtrees of order $m = \frac{1}{2}(n^2 + 1)$, $n \in 2Z^+ + 1$ form a chain of composition series

$$T(5) \subset T(13) \subset T(25) \subset \ldots \subset T(m).$$

3. Application to real world situations

3.1 Computer networking topological design

The rhomtree has strong application in star and high breed star network topological design. In this case, the root becomes the server or the heart of the network. The four edges incident on the root vertex are referred to as bridges. Other vertices are referred to as destination computers connected to the server to facilitate communication by means of four connected components of binary network of computers.

3.2 Energy resource distributions

The rhomtree as a labeled digraph has strong application in energy resource distribution network. In this case, the root vertex becomes the source vertex with the network. The four edges incident on the source vertex are referred to as bridges. Other vertices are referred to as sinks or destination vertices. This idea can be illustrated for rhomtree of order thirteen as a graphical representation of rhotrix of dimension five as shown in Fig. 4.

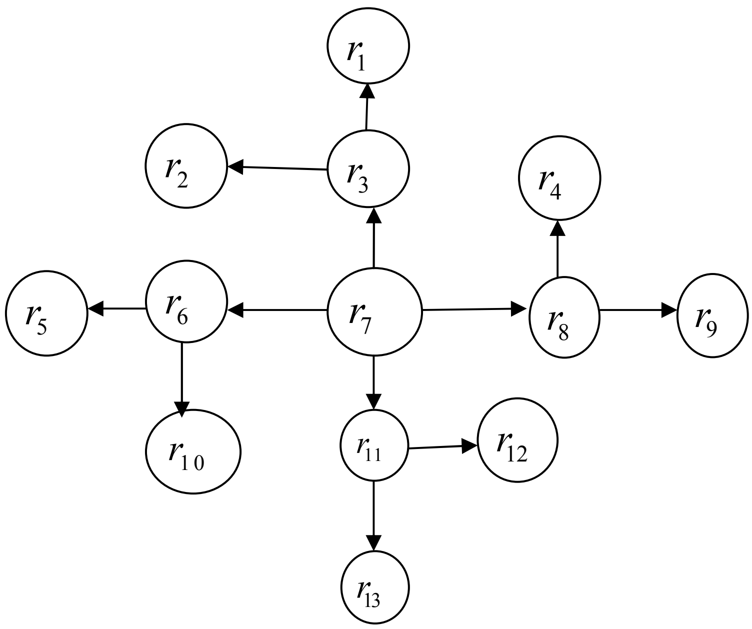


Fig. 4: A labeled directed rhomtree *T(13)* representing energy distribution network.

3.3. Other real world situations

The rhomtree of order five depicts a chemical compound called methane as shown in Fig. 5.

This methane is one of the smallest saturated hydrocarbons C_nH_{2n+2}, when enumerating their isomers with a given number *n* of carbon atoms.

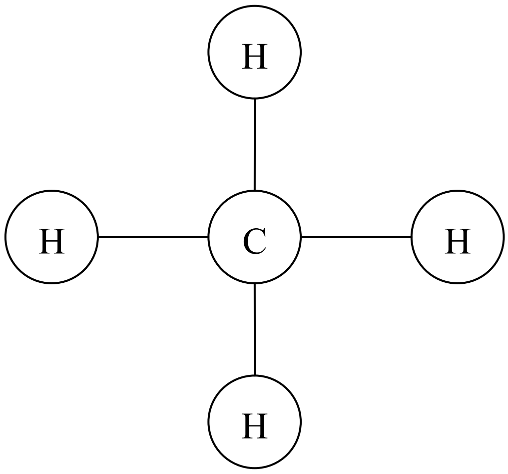


Fig. 5: Methane corresponding to rhomtree *T(5)*.

Also, the rhomtree of order thirteen represents the Cartesian product $A \times B$, of two sets $A = \{a_1, a_2, a_3, a_4\}$ and $B = \{b_1, b_2\}$ such that, *A* should consist of four elements and the set *B* should consist of two elements. Thus, $A \times B$ consists of all ordered pairs *(a, b)* where $a \in A, b \in B$. These elements of $A \times B$ can be systematically obtained by a graph of rhomtree *T(13)* shown in Fig. 6.

Observe that the root of rhomtree *T(13)* as shown in Fig.6 is labeled as $A \times B$. The first generations of the rhomtree *T(13)* are labeled as the elements of the set *A* and the second generations are labeled as the elements of the set *B*. The elements of $A \times B$ are precisely the *8* ordered pairs to the right of the rhomtree *T(13)* in Fig.6. In addition, *n(A)=4* and *n(B)=2*; hence $n(A \times B) = 8 = n(A) \cdot n(B)$.

4. Conclusion

We have initiated a concept of graphical representation of rhotrices, as objects referred to in this paper as rhomtrees. We have also shown in this work that by representing rhotrices as graphical objects, the graphical theory of rhotrix and its applications will reflect its influence in the study of multidisciplinary aspects of mathematical sciences. These include areas such as graph theory, network analysis in optimization theory and networking in computing science, which may be applied in solving real life problems. All these contribute to more areas of interest for future research directions.

5. Acknowledgement

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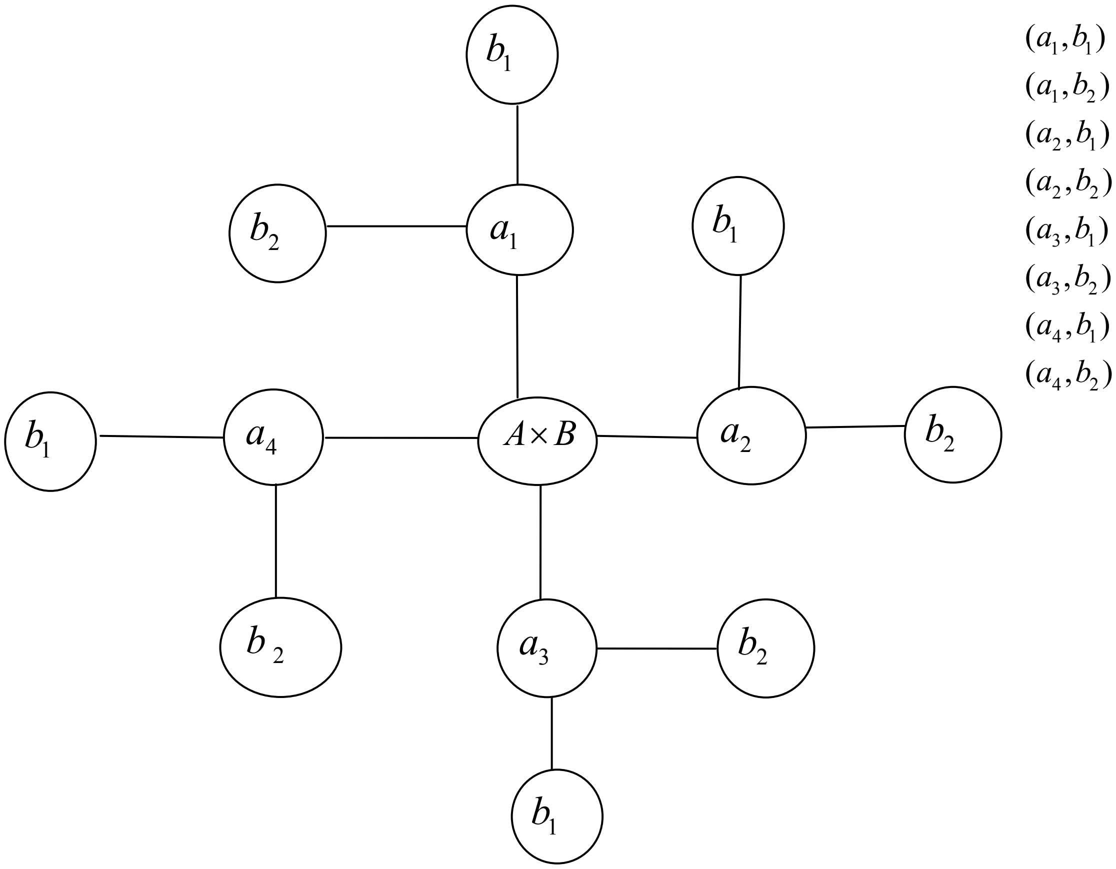

Fig. 6: Cartesian product of $A\times B=\{a_1,a_2,a_3,a_4\}\times\{b_1,b_2\}$ corresponding to rhomtree *T(13)*.

6. References

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