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A problem related to Fibonacci-like type sequences

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In [1] the digital arithmetic function ψ was defined (see also [2], where all notations used below are described).

Following [1, 2], we will mention that if the sequence $a_1, a_2, ...$ (with its members being natural numbers) is given and if

$$c_i = \psi(a_i) \ (i = 1, 2, ...),$$

then, we deduce the sequence $c_1, c_2, ...$ from the former sequence. If k and l exist so that $l \geq 0$,

$$c_{i+l} = c_{k+i+l} = c_{2k+i+l} = ...$$

for $1 \leq i \leq k$, then we shall say that

$$[c_{l+1}, c_{l+2}, ..., c_{l+k}]$$

is a *base* of the sequence $c_1, c_2, ...$ with length of k and with respect to function ψ.

Let the sequence is from Fibonacci-like type, i.e, let it have the form

$$F_0 = a, \ F_1 = b, \ F_{n+2} = F_{n+1} + F_n \ (\text{ for } n \geq 0)$$

(see, e.g., [2]).

Then we can define the function $K_S(p, q; a, b)$ that determines the length of the base of the sequence S, where a, b, p and q are natural numbers.

Open problem: For the sequence S from Fibonacci-like type, determine the values of $K_S(p, q; a, b)$ for the different values of the natural numbers a, b, p and q.

For example, as it is shown in [1], if F and L are the ordinary Fibonacci and Lucas sequences, then,

$$K_F(1, 1; 0, 1) = 24,$$

$$K_L(1, 1; 2, 1) = 24,$$

while for the extended Fibonacci sequence EF

$$F_0 = a, \ F_1 = b, \ F_{n+2} = p.F_{n+1} + q.F_n \ (\text{ for } n \geq 0)$$

it is valid

$$K_{EF}(a, b; p, q) = 216.$$

If P is the Pell-Padovan's sequence:

$$P(1) = a, \ P(2) = b, \ P(3) = c,$$

$$P(n+3) = P(n+1) + P(n), \text{ for every natural number } n \geq 1,$$

and if EP is the extended Pell-Padovan's sequence:

$$P(n+3) = pP(n+1) + qP(n), \text{ for every natural number } n \geq 1,$$

where a, b, c, p, q are real constants, then

$$K_P(a, b, c; 1, 1) = 24.$$

$$K_{EP}(a, b, c; p, q) = 936$$

(see [2, 3]).

References

[1] Atanassov, K. An arithmetic function and some of its applications. *Bull. of Number Theory and Related Topics,* Vol. IX (1985), No. 1, 18-27.

[2] Atanassov, K., D. Dimitrov and A. Shannon. A remark on ψ-function and Fibonacci sequence. *Notes on Number Theory and Discrete Mathematics* Vol. 15 (2009), No. 1, 1-10.

[3] Atanassov, K., D. Dimitrov and A. Shannon. A remark on ψ-function and Pell-Padovan's sequence. *Notes on Number Theory and Discrete Mathematics* Vol. 15 (2009), No. 2, 1-44.