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Analysis of Primes Using Right-End-Digits and Integer Structure

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Abstract

Primes were separated according to the right-end-digits (REDs) and classes in the modular ring Z_6. The primes are given by jR where j is the number of consecutive integers with RED=R (for p = 37, R = 7 and j = 3, and so on). The rows of j, in classes $\bar{2}_6, \bar{4}_6 \subset Z_6$, that contain primes, are found to have the form $\frac{1}{2}n(an \pm 1)$, a=1,3,5. A total of 499 primes were generated for n = 1 to 80 for RED=7. Similar results apply to REDs 1,3 and 9. A scalar characteristic was detected in the row structure of j.

Keywords: modular ring, triangular numbers, pentagonal numbers

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1. Introduction

The RED criteria have been found useful for the construction of spectra of various categories of integers, including primes and powers [2,3]. They have also been used in the investigation of even perfect numbers [1].

Modular rings allow primes to be sorted according to the class specification which leads to simpler analysis [2]. REDs further separate primes and hence simplify analysis even more, allowing prime sequences to be more easily extracted. Here we use the modular ring Z_6 which separates integers (Table 1). Each integer has the format $6r_i + (i-3)$ where r_i is the row and $\bar{i}_6$ the class. Note that 3|N for both $\bar{3}_6$ and $\bar{6}_6$. Furthermore, there are no even powers in $\bar{2}_6$ or $\bar{5}_6$.

Table 1

Class		$\bar{1}_6$	$\bar{2}_6$	$\bar{3}_6$	$\bar{4}_6$	$\bar{5}_6$	$\bar{6}_6$
Row	0	-2	-1	0	1	2	3
	1	4	5	6	7	8	9
	2	10	11	12	13	14	15
	3	16	17	18	19	20	21
	4	22	23	24	25	26	27

The RED separation of primes yields four sets of REDs = 1,3,7 and 9. For example, all primes ending in 7 are classified as *j*7 where *j* is the number of consecutive integers with RED=7. For primes, when $j = 0$, $p = 7$; $j = 1$, $p = 17$; and for $j = 3$, $p = 37$; and so on.

The primes are then further separated into classes $\bar{2}_6\,(6r_2 - 1)$ and $\bar{4}_6\,(6r_4 + 1)$. The rows of *j* are then analysed for sequences that will enable the primes to be predicted. Table 2 shows the classes of *j* for each type of RED. This applies to all integers in classes $\bar{2}_6$ and $\bar{4}_6$.

Table 2

Class of prime	Parity of *j*	Class of *j*			
		$R = 1$	$R = 3$	$R = 7$	$R = 9$
$\bar{4}_6$	odd	$\bar{6}_6$	$\bar{4}_6$	$\bar{6}_6$	$\bar{4}_6$
	even	$\bar{3}_6$	$\bar{1}_6$	$\bar{3}_6$	$\bar{1}_6$
$\bar{2}_6$	odd	$\bar{4}_6$	$\bar{2}_6$	$\bar{4}_6$	$\bar{2}_6$
	even	$\bar{1}_6$	$\bar{5}_6$	$\bar{1}_6$	$\bar{5}_6$

2. Integers with RED=7

Taking RED = 7 as an example, we now show how sequences of rows of *j* for primes $p = j7$ are related to quite simple sequences such as the triangular and pentagonal numbers. The row r_i yields the value of *j* according to the class $\bar{i}_6$. If, $j \in \bar{6}_6$, for example, then $j = 6r_3 + 3$, (Table 1) and *j*7yields the integer.

Primes in class $\bar{4}_6$ have *j* values as in Table3. Even $j \in \bar{3}_6(6r_3)$ and odd $j \in \bar{6}_6\,(6r_6 + 3)$(Table 2), maximum prime is 4177.

j	row	j	row	j	row	j	row	j	row
0	0	60	10	156	26	261	43	351	58
3	0	72	12	159	26	264	44	354	59
6	1	75	12	162	27	267	44	360	60
9	1	78	13	165	27	270	45	363	60
12	2	87	14	174	29	276	46	369	61
15	2	90	15	177	29	279	46	372	62
27	4	93	15	186	31	285	47	384	64
30	5	96	16	198	33	288	48	387	64
33	5	99	16	201	33	291	48	390	65
36	6	108	18	213	35	303	50	396	66
39	6	111	18	228	38	306	51	402	67
45	7	123	20	234	39	318	53	405	67
48	8	129	21	243	40	321	53	417	69
54	9	132	22	246	41	330	55		
57	9	144	24	255	42	345	57		

Table 3 lists the rows 2,7,15,26,40,57 which have the form $\frac{1}{2}n(3n+1)$, the pentagonal numbers. In fact, rows of *j* which equal such functions contain primes. Table 4 lists functions of *n* that give rows of *j* which lead to primes. The functions follow the format

$$row = \tfrac{1}{2}n(an \pm 1) \qquad (2.1)$$

with *a* = 1,3,5.

For primes in $\overline{4}_6$,

$$f(n) = \tfrac{1}{2}n(3n \pm 1),\ j \text{ odd},$$

dominates (Table 5). Primes range from 7 to 588097. Thus, commonly

$$j(even) = 6(\tfrac{1}{2}n(3n \pm 1))$$

and

$$j(odd) = 6(\tfrac{1}{2}n(3n \pm 1)) + 3$$

(Table 2). Then prime = *j*7. However, all *n* in (2.1) do not give primes when *a* constant.

Table 4: *n* values that lead to primes; RED=7, Class of prime is $\overline{4}_6$

n	***f(n)***		**Parity of *j***	**Primes $\in \overline{4}_6$**	***n***	***f(n)***		**Parity of *j***	**Primes $\in \overline{4}_6$**
	Value	Type				Value	Type		
1	2	A	**Even**	127	43	946	C	**Even**	56767
		A	**Odd**	157	44	999	C	**Even**	59407
2	7	A	**Odd**	457	45	3060	A	**Odd**	183637
3	15	A	**Even**	907	46	3197	A	**Even**	191827
		A	**Odd**	937	47	3337	A	**Even**	200227
4	26	A	**Even**	1567	48	3480	A	**Even**	208807
		A	**Odd**	1597	49	3577	B	**Odd**	214657
5	40	A	**Odd**	2437	50	3723	B	**Even**	223507
6	57	A	**Odd**	3457	51	3876	B	**Even**	232567
7	77	A	**Odd**	4657			B	**Odd**	232597
8	100	A	**Even**	6007	52	4082	A	**Odd**	244957
		A	**Odd**	6037	53	4240	A	**Even**	254407
9	117	B	**Even**	7027			A	**Odd**	254437
		B	**Odd**	7057	54	4347	B	**Odd**	260857
10	155	A	**Odd**	9337	55	7590	D	**Even**	455407
11	187	A	**Odd**	11257			D	**Odd**	455437
12	222	A	**Even**	13327	56	4732	A	**Odd**	283957
13	260	A	**Even**	15607	57	4902	A	**Even**	294127
14	301	A	**Odd**	18097			A	**Odd**	294157
15	345	A	**Even**	20707	58	5075	A	**Odd**	304537
16	392	A	**Odd**	23557	59	5251	A	**Even**	315067
17	442	A	**Odd**	26557			A	**Odd**	315097
18	477	B	**Even**	26627	60	5430	A	**Even**	325807
		B	**Odd**	26657	61	5612	A	**Even**	336727
19	532	B	**Odd**	31957			A	**Odd**	336757

20	610	A	**Even**	36607	62	9641	D	**Even**	578467
		A	**Odd**	36637			D	**Odd**	578497
21	672	A	**Odd**	40357	63	5985	A	**Odd**	359137
22	737	A	**Odd**	44257	64	6176	A	**Odd**	370597
23	805	A	**Odd**	48337	65	10595	D	**Even**	635707
24	876	A	**Even**	52567			D	**Odd**	635737
25	950	A	**Odd**	57037	66	6501	B	**Even**	390067
26	1027	A	**Even**	61627			B	**Odd**	390097
		A	**Odd**	61657	67	6767	A	**Even**	406027
27	1107	A	**Odd**	66457	69	6970	A	**Even**	418207
28	1190	A	**Odd**	71437	69	7107	B	**Even**	426427
29	1276	A	**Odd**	76597	70	7315	B	**Odd**	438937
30	1365	A	**Odd**	81737	71	7597	A	**Even**	455827
31	1457	A	**Even**	87427	72	7740	A	**Odd**	464437
32	1520	B	**Odd**	91237	73	8030	A	**Even**	481807
33	2739	D	**Odd**	164377			A	**Odd**	481837
34	1751	A	**Odd**	105097	74	8251	A	**Even**	495067
35	1855	A	**Odd**	111337	75	2850	C	**Even**	171007
36	1962	A	**Even**	117727	76	8702	A	**Even**	522127
		A	**Odd**	117757			A	**Odd**	522157
37	703	C	**Odd**	42187	77	8932	A	**Odd**	535957
38	741	C	**Odd**	44497	78	9165	A	**Odd**	549937
39	2262	B	**Even**	135727	79	9401	A	**Odd**	564097
		B	**Odd**	135757	80	9640	A	**Even**	578407
40	2420	A	**Even**	145207	81	9801	B	**Odd**	588097
41	2501	B	**Even**	150067					
		B	**Odd**	150097					
42	903	C	**Odd**	54217					

Legend: $A = \frac{1}{2}n(3n+1); B = \frac{1}{2}n(3n-1); C = \frac{1}{2}(n+1); D = \frac{1}{2}n(5n+1).$

Table 5: *n* values that lead to primes; RED=7, Class of primes is $\bar{4}_6$

$f(n)$ = row of *j*	$n : j(even) = 6f(n)\ (\bar{3}_6)$	$n : j(odd) = 6f(n)+3\ (\bar{6}_6)$
$\frac{1}{2}(n+1)$	1,3,4,5,9,10,11,15,16,17,18,22, 23,25,26,29,32,37,39,40,43,44, 50,54.58.59.60.66.75	1,3,5,6,14,15,19,20,22,27,38,42,45,50, 54,63,66,68
$\frac{1}{2}n(3n+1)$	1,3,4,8,12,13,15,20,24,26,31,36, 40,46,47,48,53,57,59,60,61,67, 71,73,74,76,80	1,2,3,4,5,6,7,8,10,11,14,16,17,20,21,22, 23,25,26,27,28,29,30,34,35,36,45,47,48, 52,53,56,57,58,59,61,63,64,73,76,77,78, 79
$\frac{1}{2}n(3n-1)$	1,2,3,4,6,8,9,10,11,13,16,18,20, 29,30,41,45,46,50,51,52,57,58, 59,64,66,69,74	1,2,3,5,8,9,10,11,12,14,17,18,19,20,21,2 2, 23,24,29,31,32,36,38,39,40,41,48,49,51, 53,54,57,58,59,60,61,66,67,68,70,72,76, 77,78,79,80
$\frac{1}{2}n(5n+1)$ *	3,5,10,13,17,19,20,37,38,40,41, 44,47,55,58,59,62	4,9,11,12,16,18,20.23,26,30,33,35,41,46, 51,55,60,61,62
$\frac{1}{2}n(5n-1)$ *	1,2,4,5,6,9,11,12,13,20,22,29, 36,47,48,57	1,2,3,5,7,8,10,12,14,16,19,21,23,24,28,3 1, 36,38,40,47,51,54

*maximum *n* = 62

Similar results are obtained for primes in Class $\bar{2}_6$, except that in this case the dominant {*f*(*n*)} are the triangular numbers, $\frac{1}{2}n(n+1)$, *j* even. Table 6 displays the *n* values for various *f*(*n*) which lead to $\bar{2}_6$ class primes.

Table 6: *n* values that lead to primes; RED=7, Class of primes is $\bar{2}_6$

$f(n)$ = row of *j*	$n : j(even) = 6f(n)-2\ (\bar{1}_6)$	$n : j(odd) = 6f(n)+1\ (\bar{4}_6)$
$\frac{1}{2}(n+1)$	1,2,3,4,5,7,9,11,14,15,16,18,23,	1,2,4,6,7,13,14,21,30,37,39,46,49,58,59,

	24,32,33,41,42,43,44,50,54,55, 56,58,59,62,63,66,69,70,72,74, 78	63,65,69,79
$\frac{1}{2}n(3n+1)$	1,3,6,8,9,14,20,22,28,31,45,48, 50,51,59,64,66,73	1,5,7,9,12,14,15,16,18,26,33,35,37,39, 46,47,49,56,58,67,70,72,75,77
$\frac{1}{2}n(3n-1)$	1,4,5,8,14,18,19,21,25,33,34,36, 40,49,54,55,59,63,67,68,69,70, 71,76,77	2,7,24,26,27,28,31,37,38,41,48,52,62, 65,66,75,76
$\frac{1}{2}n(5n+1)$ *	1,2,3,7,10,16,23,24,30,37,38,41, 42,43,53,54,56,58,59,61	1,2,5,7,8,9,12,14,15,18,20,21,22,26,28, 29,33,36,37,48,49,52,58,59,60,62
$\frac{1}{2}n(5n-1)$ *	1,4,7,9,11,12,15,16,20,24,25,28, 30,35,38,43,44,49,50,55,58,62	1,2,3,4,5,6,8,9,10,11,13,15,19,20,21,27, 32,36,45,46,47,48,49,50,53,56,57,61

*maximum n = 62

3. Integers with RED=1

For this RED and Class $\bar{4}_6$, the rows which contain primes have the same pattern as for R=7. A sample for the first twenty n is shown in Table 7. Odd j for $f(n)=\frac{1}{2}n(5n+1)$ yields the most primes.

Table 7: n values that lead to primes; RED=1, Class of primes is $\bar{4}_6$

$f(n)$ = row of j	$n: j(even)=6f(n)\ (\bar{3}_6)$	$n: j(odd)=6f(n)+3\ (\bar{6}_6)$
$\frac{1}{2}(n+1)$	1,2,4,8,10,14,16,17,20	4,6,9,10,16
$\frac{1}{2}n(3n+1)$	2,7,8,9,13,14	1,7,8,9,11,15,19
$\frac{1}{2}n(3n-1)$	1,4,6,7,8,12,13,15,18,20	2,3,5,7,10,13
$\frac{1}{2}n(5n+1)$	1,2,4,6,7,9,11,12,13,19,20	1,2,3,4,5,7,8,9,10,12,13,14,15,17,18,19, 20
$\frac{1}{2}n(5n-1)$	2,4,15,17,18	1,2,3,4,5,8,10,11,15,16,19

Similar results apply for Class $\bar{2}_6$. n values applicable to the various row functions are set out in Table 8.

Table 8: n values that lead to primes; RED=1, Class of primes is $\bar{2}_6$

$f(n)$ = row of j	$n: j(even) = 6f(n) - 2\ (\bar{1}_6)$	$n: j(odd) = 6f(n) + 1\ (\bar{4}_6)$
$\frac{1}{2}(n+1)$	1,5,8,13,17	1,2,5,9,12,13,14,15,16,18,19,20
$\frac{1}{2}n(3n+1)$	1,2,3,5,8,9,10,13,14,15,17	1,2,3,4,5,8,10,12,16,19
$\frac{1}{2}n(3n-1)$	1,3,4,5,6,8,9,10,16,20	1,2,5,7,8,12,13,14,16,18
$\frac{1}{2}n(5n+1)$ *	2,5,7,9,11,16,18	1,3,4,5,6,8,15,16,20
$\frac{1}{2}n(5n-1)$ *	1,2,5,7,8,12,14,17	1,6,7,8,13,15,19

*maximum n = 62

4. Integers with RED=3 or 9

This section applies to Classes $\bar{2}_6$ and $\bar{4}_6$. The row that contain primes have the form

$$\tfrac{1}{2}n(an \pm 1), a \in \{1,3,5\}.$$

Table 9 lists n values that lead to primes in Class $\bar{4}_6$, while Table 10 lists n values that lead to primes in Class $\bar{2}_6$. $P = j3$ or $j9$ in both tables.

Table 9: n values that lead to primes; p = *j3 or j*9, Class of primes is $\bar{4}_6$

$f(n)$ = row of j	RED	$n: j(even) = 6f(n) - 2\ (\bar{1}_6)$	$n: j(odd) = 6f(n) + 1\ (\bar{4}_6)$
$\frac{1}{2}(n+1)$	3	1,2,5,7,8,9,11,12,13,18,19,20	1,2,3,4,7,9,10,15,18,20
	9	3,6,7,9,13,20	1,2,3,4,5,6,7,8,9,10,13,15,16,17,20
$\frac{1}{2}n(3n+1)$	3	1,4,5,7,10,13,14,18,20	2,6,9,19
	9	1,2,4,5,9,12,14,16,19	1,2,3,4,7,10,11,12,13,15,16,17
$\frac{1}{2}n(3n-1)$	3	1,2,4,5,8,12,14,16,18	1,2,3,5,10,12,15,16
	9	3,5,6,10,12,14,16	1,3,6,7,9,10,12,14,15,20

$\frac{1}{2}n(5n+1)*$	3	1,2,3,4,6,12,14,15,18	1,2,3,7,9,10,11,14,15
	9	3,5,6,7,8,9,10,19,20	1,3,4,5,8,10,13
$\frac{1}{2}n(5n-1)*$	3	1,2,5,8,9,10,11,13	4,5,6,10,17,18,19
	9	1,3,4,6,7,8,11,15,17,18,19	1,3,7,11,13,17,18,20

Table 10: *n* values that lead to primes; $p = j3$ or $j9$, Class of primes is $\bar{2}_6$

$f(n)$ = row of j	RED	$n : j(even) = 6f(n) + 2\ (\bar{5}_6)$	$n : j(odd) = 6f(n) - 1\ (\bar{2}_6)$
$\frac{1}{2}(n+1)$	3	1,3,6,10,12,13,14,19	1,2,3,4,8,9,12,15,18,19
	9	1,3,5,6,7,9,10,11,14,15,17,18	1,2,3,4,6,9,10,12,14,18,19
$\frac{1}{2}n(3n+1)$	3	2,4,5,7,9,10,11,18	1,4,5,6,10,11,12,15,17
	9	1,2,3,6,7,8,9,13,14,16,20	2,4,5,9,14,20
$\frac{1}{2}n(3n-1)$	3	1,3,6,9,13,17,18,20	1,2,10,13,15,20
	9	1,5,6,7,11,18	1,3,4,5,8,9,10,11,18
$\frac{1}{2}n(5n+1)*$	3	2,4,5,7,11,13,16,20	1,2,3,6,8,9,13,16,17,19
	9	4,5,7,10,17,18	1,2,3,7,8,10,14,18
$\frac{1}{2}n(5n-1)*$	3	2,3,5,9,10,17	1,4,5,13,20
	9	1,2,3,4,9,12,13,14,15,19	3,10,14,17,20

5. Final Comments

For R=7, with n =1 to 80, the five $f(n)$ displayed in Table 11 generated nearly 500 primes. It is noteworthy that, while, as expected, the triangular numbers and the pentagonal numbers appear in Sloane and Ploufe [4], the other sequences in Table 11 are not to be found there even though their formulations are so simple. As can be seen, the parity and class structure criteria illuminate some aspects of the complexity of the primes.

Table 11: n values of $f(n)$ that lead to primes; RED=7, Class of prime is $\bar{4}_6$

$f(n)$	***n* = 1,2,3, …, 20**
$\frac{1}{2}(n+1)$	1,3,6,10,15,21,28,36,45,55,66,78,91,105,120,136,153,171,190,210
$\frac{1}{2}n(3n+1)$	2,7,15,26,40,57,77,100,126,155,187,222,260,301,345,392,442,495,551,610
$\frac{1}{2}n(3n-1)$	1,5,12,22,35,51,70,92,117,145,176,210,247,287,330,376,425,477,532,590
$\frac{1}{2}n(5n+1)*$	3,11,24,42,65,93,126,164,207,255,308,366,429,497,570,648,731,819,912,1010
$\frac{1}{2}n(5n-1)*$	2,9,21,38,60,87,119,156,198,245,297,354,416,483,555,652,714,801,893,990

A comparison of these values in Table 11 with the n values set out in Tables 5 and 6 shows the embedded sequences which follow $f(n)$. For example, in Table 5 (Class $\bar{4}_6$), with j odd the triangular numbers are embedded in the sequence in the second row and the pentagonal numbers are also embedded in the sequence in the third row. This suggests further investigation of a possible fractal quality to the n series.

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