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AN INTERPRETATION OF PASCUAL–LEONE'S DISCRETE MATHEMATICAL MODEL OF HUMAN INFORMATION PROCESSING CAPACITY

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Abstract

This paper outlines the pioneering work of Juan Pascal-Leone's investigations of the cognitive load on learners. It utilizes a recursive scheme in a Piagetian cognitive development framework to measure the maximum number of discrete "chunks" of information or schemes that our working memory can process, control or integrate at any one time. Pascual-Leone further designed a new compound-stimuli visual information type of task for testing his theories quantitatively. The stochastic model used for his predictions was the Bose-Einstein occupancy model of combinatorial analysis.

Keywords: Piaget, Bose-Einstein, schemas, cognitive load, schemata, motoric responses, attending acts, set measure.

1. Introduction

"Learning is defined as change in long-term memory. In recent years psychologists have sought to understand learning and the limitations that are imposed on learning in the classroom environment. One strong recommendation in recent research in cognitive psychology is that teachers are able to facilitate schema acquisition and schema automation for the student by minimising any cognitive load that may occur" [3]. Pascual–Leone [12] has developed a model for information-processing capacity associated with memory and utilized in problem-solving. His theoretical predictions accord well with experimental measurements of a relational system, though his ideas have not received the recognition they deserve, perhaps because they did not accord with the *zeitgeist* in the manner of Piaget [13].

This system corresponds to the Piagetian notion of a 'field of equilibrium' and is the developmental change of the attention span [7]. Indeed, Pascual-Leone has proposed an important corollary to the Piagetian view of inferential processes [8]. His suggestion of a numerical characteristic for each Piagetian general stage helps us to understand the transition from one stage to the next. This numerical characteristic can be interpreted as the set measure of the Piagetian field of equilibrium, and it is the number of separate schemas or separate chunks of information on which someone can simultaneously operate with his mental structures.

Schemata play an important role in storing and organising information. When students interact with the world, their experiences are stored and organised into mental structures known as schemas. Meaning is assigned to stimuli, capturing common properties of specific behaviours, objects and experiences [14]. Though much of the empirical work has been carried out with school students, the tertiary teachers' work can be helped if they are aware of what stage of psychological development their students are at. Moreover, their teaching can be more effective if they treat the adult learner as proceeding through analogous development stages, hopefully rapidly, when encountering new ideas or solving problems. These issues seem to transcend cultural boundaries [6].

2. Pascual-Leone's Approach

Pascual-Leone's theory concerns the first two of the following three components of the student viewed as a relatively autonomous psychological system:

- a repertoire, *H*, of behavioural schemes;
- a central processing space, *M*;
- psychological laws such as learning laws and field organisation laws.

Essentially *H* is the memory, and the *M* space is where the information processed by the activated schemes $H^* \subset H$ is transformed or integrated into novel behaviour. Pascual-Leone further distinguishes between

- the subject's maximum *M* capacity or 'structural *M*', M_s and
- his/her 'functional *M*', M_f, the amount of *M* space actually used by them at any particular moment of their cognitive activity.

As *H* changes with learning so will the level of performance even if the subject's M value remains invariant. This can account for both response variability and the general structural invariants of the Piagetian stages which are seen as qualitative manifestations of this hidden parameter *M*. To describe the inter-relation of *H* and *M* we begin with *n* simple cues, $S_1, S_2 ..., S_n$ which collectively constitute a compound stimulus

$$S^n = \{S_1, S_2, ..., S_n\}. \tag{2.1}$$

These activate a relevant perceptual subschema

$$H_s^* = \{z_1, z_2, ..., z_n\}. \tag{2.2}$$

There are two other schemas, ψ_I and ψ_S, which represent the task instructions and the general task situation, respectively. These special schema are activated throughout the cognitive process, and they, as it were, determine the 'program' to be followed. The subject has a number of discrete 'energy' units to 'turn on' or increase the level of activity of one or more of the z_i. It is supposed that *a* energy units are expended on the special schema and the and the remaining *k* = *m* - *a* are available for the elements of H_s^*. Each one of these *k* energy units is randomly applied to one of the z_i, simultaneously with, and independently from, all the other *k*-1 available energy units. This application of the *m* equal and indistinguishable energy units to ψ_I, ψ_S and H_S^* is called an 'attending act', A_j .

The outcome of any attending act A_j will be a set of *u* of the z_i schemes at varying degrees of intensity, where

$$u \in \{1,2,...,k\},$$

and

$$i \in \{1,2,...,n\}.$$

$$u = u\left(c_j x_j f_j^*\right)$$

in which

- c_j, the *content* of A_j, is the actual list of schema activated by A_j;
- x_j, the *number* of z_i activated by A_j;
- f_j^*, the numerical *function* which assigns to every element in c_j a natural number to indicate the number of energy units from M which have energized the corresponding schema.

If

$$A = \{A_1 A_2, ..., A_k\},$$

then

$$A = A\left(, X, F^*\right),$$

where

$$C = \{c_1 c_2, ..., c_k\} \tag{2.3}$$

corresponds to the concept of '*content* of attention',

$$X = \{x_1 x_2, ..., x_k\} \tag{2.4}$$

corresponds to '*span* of attention', and

$$F^* = \{f_1^*, f_2^*, ..., f_k^*\} \tag{2.5}$$

corresponds to '*intensity* of attention'.

3. Motoric Responses

Pascual-Leone measured, x_j, the number of different motoric responses of the form $R\left(z_j\right)$, which the subject produces after each attending act, A_j, so that x_j is thus an empirical correlate of the concept X which is used as the dependent variable in the following recursive scheme.

The first attending act A_1, produces x_1 motoric responses which then alter the content of H to include a new scheme, $\Phi\left(Ax_1\right)$. The subject is then assumed to evaluate his Compound-Stimuli Visual Information Task (CSVI) processing by attending to (activating) the schemes ψ_I, ψ_S and $\Phi\left(Ax_1\right)$. which represent the previous activities. Thus, unless some of the remaining k-1 M units are redundantly applied to these three schemes, the 'unsaturatedness' of M allows the subject to start a new attending act, A_2. This recursive cycle progresses through k successive attending acts.

The empirical dependent variable

$$x = x_1 + x_2 + ... + x_k \tag{3.1}$$

of different and relevant motoric responses produced by the subject by the end of this process has a range from 1 to n, although $0 \le x_j \le n$ since the zero value could arise if the subject had exhausted the set of relevant responses which could be produced in relation to S^n.

It is assumed that the process, A_j, by which the response, x_j, is produced is invariant. If S^n and H_S^* are invariant for successive A_j belonging to the same item, and if the result of these successive A_j is cumulative for any item, then the total outcome of $A_1 + A_2 + \ldots + A_k$ should be the same as the outcome of a single A_j in which the number of energy units utilized is equal to the sum of the energy units used in each one of the A_j. Since in any A_j the number of energy units available is *k*, the total number of different energy units used by the subject at the end of the attending process will be k^2. Thus the random variable *X* depends upon the two parameters *k* and *n* (the number of simple stimuli included in the compound S^n being shown to the subject and represented by the subject in H_S^*.

4. Results

Pascual-Leone applied Bose-Einstein statistics [4] to develop a combinatorial occupancy model. In this sampling scheme, the probability of any distinguishable arrangement is equal to

$$\frac{1}{\binom{n+k-1}{k}}$$

in which

- *n* is the number of cells available (that is, the number of cues in the stimulus compound S^n), and
- *k* is the number of energy units which energise schemes other than ψ_I and ψ_S.

It is assumed that the subschemes of *H* are equiprobable and that the energy units are indistinguishable, so that the act of energizing a scheme corresponds to randomly assigning k^2 energy units among a schema. The numbers of different relevant responses produced by the subject corresponds to the number of energized schema [9]. The probability that at the end of his attending process a subject will have responded to *x* different cues from S^n is

$$\Pr(x) = \frac{\binom{n}{n-x}\binom{k^2-1}{x-1}}{\binom{n+k^2-1}{k^2}} \tag{4.1}$$

The values for k in Pascual-Leone's experiments are set out in Table 1:

age (years)	5-6	7-8	9-10	11-12
k	2	3	4	5

Table 1: Elements of the perceptual subscheme

His experiments contrasted S^n with R^j, *j* = 1,2, …, *n* where R^j indicates *j* acceptable values taken by *X*. These results accord with those of Miller [9] who described the upper limit of the information-processing capacity of adults as the 'magical number seven'.

5. Discussion

Cognitive Load is anything that does not assist change to long-term memory; that is, it has the ability to hinder schema construction. Cognitive Load theory argues that some learning environments impose greater demands than others. As the amount of information increases, so too does the strain on the associated cognitive functions [11]. Cognitive overload occurs when the demands of a task exceed the capabilities of working memory. No learning can occur during cognitive overload. Chandler and Sweller [2], two of the major figures in cognitive load research, have stated that optimum learning occurs when working memory load is kept to a minimum. This best facilitates the changes in long-term memory.

There are three types of cognitive load, extraneous, intrinsic and germane.

- *extraneous load* results from the manner in which the to-be-learned information is presented to the learner;
- *intrinsic load* is caused by the inherent properties of the to-be-learned information. This can be measured in element interactivity: independent elements of information that need to be processed simultaneously for understanding;
- *germane load* is cognitive load devoted to schema construction and automation by utilising free working memory capacity [10].

The point of being aware of the maximum number of chunks of information that can be handled is that it is important for a teacher to minimise extraneous and intrinsic load so that working memory can allocate more resources to schema formation, thus facilitating problem solving [5] and abstraction [1].

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