NNTDM 13 (2007) 4, 11-20

On some Pascal's like triangles. Part 4

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In a series of papers, starting with [1, 2, 3], we discuss new types of Pascal's like triangles. Triangles of the present form, but not with the present sense, are described in different publications, e.g. [4, 5, 6], but at least the author had not found a research with similar idea.

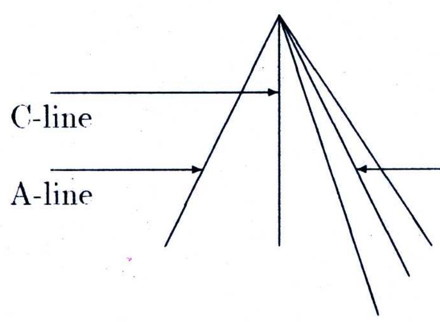
In the first part of our research we studied properties of some standard sequences; in the second part – of some "special" sequences; in the third part we construct $(0, 1)$-analogous of the Pascal's like triangles (or "(mod 2)-triangles") from the both previous papers, i.e., we constructed (*mod* 2)-values of their elements and discussed the obtained configurations.

All these triangles are planar objects. Here, we shall construct 3-dimensional analogues of the most important from the so constructed triangles.

We shall construct infinite pyramids following the ideas from [1, 2, 3]. We shall show the elements of k-th level as values in a $((2k+1) \times (2k+1))$-square, because the pyramid picture is not visuable enough.

Let us note the following three important lines of the (infinite) pyramid as follows (see the figure below):

• A-lines - lines that correspond to the four pyramid edge,
• B-lines - lines that correspond to the four (infinite) "bisectors" of the infinite triangle with two sides - the two pyramid edges,
• C-line - the line that corresponds to the (infinite) pyramid "altitude".



First, we construct the first four levels of a pyramid with given elements $a, b, c, \ldots$ lying on its A-lines.

a

b	$a+b$	b
$a+b$	$2a+b$	$a+b$
b	$a+b$	b.

c	$b+c$	$a+b+c$	$b+c$	c
$b+c$	$2b+c$	$2a+3b+c$	$2b+c$	$b+c$
$a+b+c$	$2a+3b+c$	$4a+4b+c$	$2a+3b+c$	$a+b+c$
$b+c$	$2b+c$	$2a+3b+c$	$2b+c$	$b+c$
c	$b+c$	$a+b+c$	$b+c$	c

Because the next squares of this pyramid are large, below we show only the left part (i.e., with dimension 4×7) for square 7×7.

d	$c+d$	$b+2c+d$	$a+3b+3c+d$	...
$c+d$	$2c+d$	$2b+3c+d$	$2a+5b+4c+d$	...
$b+2c+d$	$2b+3c+d$	$4b+4c+d$	$4a+8b+5c+d$	...
$a+3b+3c+d$	$2a+5b+4c+d$	$4a+8b+5c+d$	$8a+12b+6c+d$	...
$b+2c+d$	$2b+3c+d$	$4b+4c+d$	$4a+8b+5c+d$	...
$c+d$	$2c+d$	$2b+3c+d$	$2a+5b+4c+d$	...
d	$c+d$	$b+2c+d$	$a+3b+3c+d$	...

Second, we construct the first three levels of a pyramid with given elements $a, b, c, \ldots$ lying on its B-lines.

a

$-a+b$	b	$-a+b$
b	$a+b$	b
$-a+b$	b	$-a+b$

$a-2b+c$	$-b+c$	c	$-b+c$	$a-2b+c$
$-b+c$	$-a+c$	$b+c$	$-a+c$	$-b+c$
c	$b+c$	$a+2b+cc$	$b+c$	c
$-b+c$	$-a+c$	$b+c$	$-a+c$	$-b+c$
$a-2b+c$	$-b+c$	c	$-b+c$	$a-2b+c$

Third, we construct the first three levels of a pyramid with given elements $a, b, c, \ldots$ lying on its C-line.

a

$-a$	$-a+b$	$-a$
$-a+b$	b	$-a+b$
$-a$	$-a+b$	$-a$

$3a-3b+c$	$2a-3b+c$	$a-2b+c$	$2a-3b+c$	$3a-3b+c$
$2a-3b+c$	$a-3b+c$	$-b+c$	$a-3b+c$	$2a-3b+c$
$a-2b+c$	$-b+c$	c	$-b+c$	$a-2b+c$
$2a-3b+c$	$a-3b+c$	$-b+c$	$a-3b+c$	$2a-3b+c$
$3a-3b+c$	$2a-3b+c$	$a-2b+c$	$2a-3b+c$	$3a-3b+c$

Now, we shall give some particular cases.

First, we construct a pyramid with A-lines, all elements of which are "1". Obviously, we can interprete these elements as the sequential powers of "1".

1

1	2	1
2	3	2
1	2	1

1	2	4	2	1
2	3	6	3	2
4	6	9	6	4
2	3	6	3	2
1	2	4	2	1

1	2	4	8	4	2	1
2	3	6	12	6	3	2
4	6	9	18	9	6	4
8	12	18	27	18	12	8
4	6	9	18	9	6	4
2	3	6	12	6	3	2
1	2	4	8	4	2	1

Therefore (cf. [1]) we see that the elements of the B-lines are the sequential powers of "2" and of the C-lines – the sequential powers of "3". By analogy with [1] we can generalize this pyramid for case: A-lines contain elements of the sequential powers of n, B-lines – of $n + 1$ and C-elements – of $n + 2$. For example, when A-lines contain elements of the sequential powers of 2, then B-lines contain the sequential powers of "3" and C-elements – of "4".

1

2	3	2
3	4	3
2	3	2

4	6	9	6	4
6	8	12	8	6
9	12	16	12	9
6	8	12	8	6
4	6	9	6	4

8	12	18	27	18	12	8
12	16	24	36	24	16	12
18	24	32	48	32	24	18
27	36	48	64	48	36	27
18	24	32	48	32	24	18
12	16	24	36	24	16	12
8	12	18	27	18	12	8

Second, we construct a pyramid with A-lines which elements are the sequential natural numbers.

1

2	3	2
3	4	3
2	3	2

3	5	8	5	3
5	7	11	7	5
8	11	15	11	8
5	7	11	7	5
3	5	8	5	3

4	7	12	20	12	7	4
7	10	19	28	19	10	7
12	19	26	39	26	19	12
20	28	39	54	39	28	20
12	19	26	39	26	19	12
7	10	19	28	19	10	7
4	7	12	20	12	7	4

If we construct a pyramid with B-lines which elements are the sequential natural numbers, it will have the form

1

1	2	1
2	3	2
1	2	1

0	1	3	1	0
1	2	5	2	1
3	5	8	5	3
1	2	5	2	1
0	1	3	1	0

0	0	1	4	1	0	0
0	1	2	7	2	1	0
1	2	4	12	4	2	1
4	7	12	20	12	7	4
1	2	4	12	4	2	1
0	1	2	7	2	1	0
0	0	1	4	1	0	0

If we construct a pyramid with C-line which elements are the sequential natural numbers, we shall obtain the pyramid:

1

0	1	0
1	2	1
0	1	0

−1	−1	0	−1	−1
−1	−1	1	−1	−1
0	1	3	1	0
−1	−1	1	−1	−1
−1	−1	0	−1	−1

2	1	0	0	0	1	2
1	0	−1	0	−1	0	1
0	−1	−2	1	−2	−1	0
0	0	1	4	1	0	0
0	−1	−2	1	−2	−1	0
1	0	−1	0	−1	0	1
2	1	0	0	0	1	2

Finally, following the idea for $(0,1)$-triangles from [3], we shall construct analogous of the above digital pyramids.

For the first above pyramid we obtain its $(0,1)$-analogous with form:

1

1	0	1
0	1	0
1	0	1

1	0	0	0	1
0	1	0	1	0
0	0	1	0	0
9	1	0	1	0
1	0	0	0	1

1	0	0	0	0	0	1
0	1	0	0	0	1	0
0	0	1	0	1	0	0
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	1	0	0	0	1	0
1	0	0	0	0	0	1

For second pyramid we obtain:

1

0	1	0
1	0	1
0	1	0

0	0	1	0	0
0	0	0	0	0
1	0	0	0	1
0	0	0	0	0
0	0	1	0	0

0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0

For third pyramid we have:

1

0	1	0
1	0	1
0	1	0

1	1	0	1	1
1	1	1	1	1
0	1	1	1	0
1	1	1	1	1
1	1	0	1	1

0	1	0	0	0	1	0
1	0	1	0	1	0	1
0	1	0	1	0	1	0
0	0	1	0	1	0	0
0	1	0	1	0	1	0
1	0	1	0	1	0	1
0	1	0	0	0	1	0

Fourth pyramid has $(0,1)$-form:

1

0	1	0
1	0	1
0	1	0

0	0	1	0	0
0	0	0	0	0
1	0	0	0	1
0	0	0	0	0
0	0	1	0	0

0	0	0	1	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
1	0	0	0	0	0	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	1	0	0	0

Fifth pyramid has $(0,1)$-form:

1

0	1	0
1	0	1
0	1	0

1	1	0	1	1
1	1	1	1	1
0	1	1	1	0
1	1	1	1	1
1	1	0	1	1

0	1	0	0	0	1	0
1	0	1	0	1	0	1
0	1	0	1	0	1	0
0	0	1	1	1	0	0
0	1	0	1	0	1	0
1	0	1	0	1	0	1
0	1	0	0	0	1	0

In conclusion we shall mention that the modifications of Pascal's triangle, discussed in the four parts of the research are only some of the possible. In next research we shall construct some other kinds of Pascal's triangle modifications. All these new objects generate new mathematical ideas and constructions. For example, the already constructed triangles and pyramids show the possibility for introducing of new concepts related to numerical sequences, that will be an object of a future research.

References

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