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ON TWO NEW 2-FIBONACCI SEQUENCES

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To Prof. Ivan Dotsinsky
for his 70-th birthday!

Let the arbitrary real numbers a, b, c, and d be given.

In [1, 2] four different ways of constructing two sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ are described. We shall call them 2-Fibonacci sequences (or 2-F-sequences). The four schemes are the following

$$\begin{array}{l} \alpha_0 = a, \ \beta_0 = b, \ \alpha_1 = c, \ \beta_1 = d \\ \alpha_{n+2} = \beta_{n+1} + \beta_n, \ n \geq 0 \\ \beta_{n+2} = \alpha_{n+1} + \alpha_n, \ n \geq 0 \end{array} \quad (1)$$

$$\begin{array}{l} \alpha_0 = a, \ \beta_0 = b, \ \alpha_1 = c, \ \beta_1 = d \\ \alpha_{n+2} = \alpha_{n+1} + \beta_n, \ n \geq 0 \\ \beta_{n+2} = \beta_{n+1} + \alpha_n, \ n \geq 0 \end{array} \quad (2)$$

$$\begin{array}{l} \alpha_0 = a, \ \beta_0 = b, \ \alpha_1 = c, \ \beta_1 = d \\ \alpha_{n+2} = \beta_{n+1} + \alpha_n, \ n \geq 0 \\ \beta_{n+2} = \alpha_{n+1} + \beta_n, \ n \geq 0 \end{array} \quad (3)$$

$$\begin{array}{l} \alpha_0 = a, \ \beta_0 = b, \ \alpha_1 = c, \ \beta_1 = d \\ \alpha_{n+2} = \alpha_{n+1} + \alpha_n, \ n \geq 0 \\ \beta_{n+2} = \beta_{n+1} + \beta_n, \ n \geq 0 \end{array} \quad (4)$$

In [1] L. Atanassova, D. Sasselov, and the author discussed scheme (1), while scheme (2) has been studied by the author in [2] (see, also [3-9]).

Graphically, the (n+2)-nd members of the different schemes are obtained from the n-th and the (n+1)-st members as shown in Fig. 1-4.

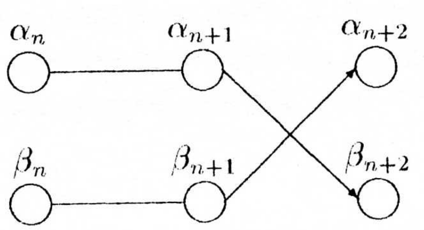

Fig. 1.

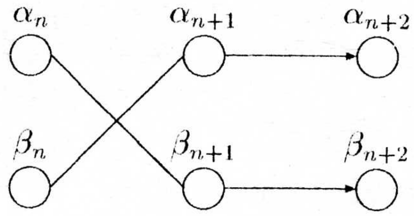

Fig. 2.

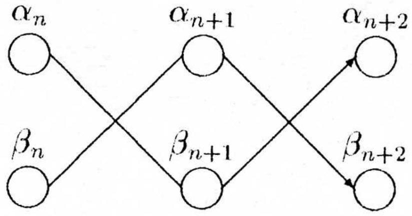

Fig. 3.

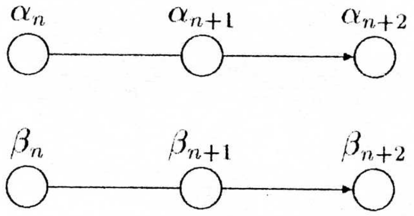

Fig. 4.

Clearly, if we set $a = b$ and $c = d$, then sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$ will coincide with each other and with the sequence $\{F_i\}_{i=0}^{\infty}$, which is called a generalized Fibonacci sequence, where

$$F_0(a,c) = a, \ F_1(a,c) = c, \ F_{n+2}(a,c) = F_{n+1}(a,c) + F_n(a,c).$$

Let $F_i = F_i(0,1)$; $\{F_i\}_{i=0}^{\infty}$ be the ordinary Fibonacci sequence.

Now, we shall introduce two new (mixed) schemes, which have the following recurrence forms:

$$\begin{aligned} &\alpha_0 = a, \ \beta_0 = b, \ \alpha_1 = c, \ \beta_1 = d \\ &\alpha_{2n+2} = \beta_{2n+1} + \beta_{2n} \\ &\beta_{2n+2} = \alpha_{2n+1} + \alpha_{2n} \\ &\alpha_{2n+3} = \alpha_{2n+2} + \beta_{2n+1} \\ &\beta_{2n+3} = \beta_{2n+2} + \alpha_{2n+1} \\ &(n \geq 0) \end{aligned} \tag{5}$$

and

$$\begin{array}{l} \gamma_0 = a,\ \delta_0 = b,\ \gamma_1 = c,\ \delta_1 = d \\ \gamma_{2n+2} = \gamma_{2n+1} + \delta_{2n} \\ \delta_{2n+2} = \delta_{2n+1} + \gamma_{2n} \\ \gamma_{2n+3} = \delta_{2n+2} + \delta_{2n+1} \\ \delta_{2n+3} = \gamma_{2n+2} + \gamma_{2n+1} \\ (n \geq 0) \end{array}, \tag{6}$$

and the graphical interpretations from Fig. 5 and 6.

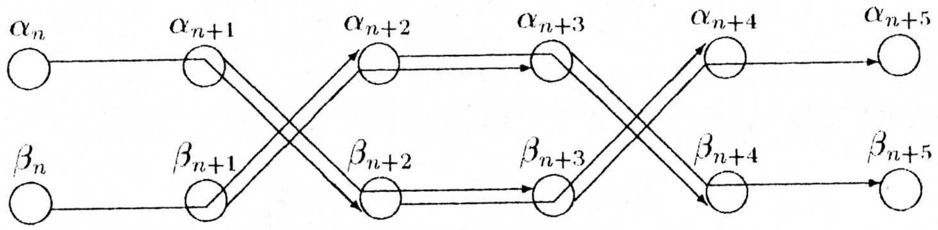

Fig. 5.

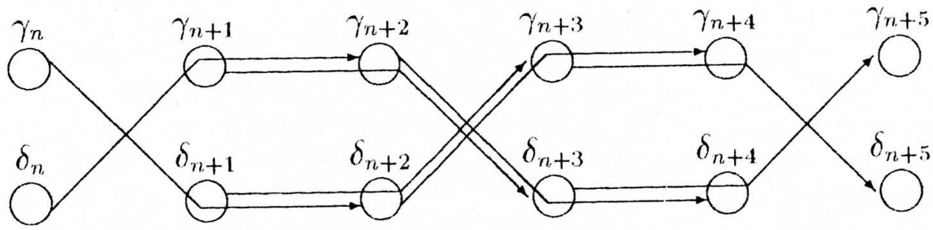

Fig. 6.

The first ten terms of the sequences defined in (5) are:

n	α_n	β_n
0	a	b
1	c	d
2	$b + d$	$a + c$
3	$b + 2d$	$a + 2c$
4	$2a + 3c$	$2b + 3d$
5	$3a + 5c$	$3b + 5d$
6	$5b + 8d$	$5a + 8c$
7	$8b + 13d$	$8a + 13c$
8	$13a + 21c$	$13b + 21d$
9	$21a + 34c$	$21b + 34d$

while these, for the sequences defined in (6) are: