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RESTRICTIVE FACTOR: DEFINITION, PROPERTIES AND PROBLEMS

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The paper is a continuation of [1,2], where the concepts of the irrational factor and the converse factor have been introduced, respectively, and the used notations are described.

Every natural number n has a canonical representation in the form $n = \prod_{i=1}^{k} p_i^{\alpha_i}$, where $p_1, p_2, ..., p_k$ are different prime numbers and $\alpha_1, \alpha_2, ..., \alpha_k \geq 1$ are natural numbers.

In [1,2] they are juxtaposed to n the (real) numbers

$$IF(n) = \prod_{i=1}^{k} p_i^{1/\alpha_i}$$

and

$$CF(n) = \prod_{i=1}^{k} \alpha_i^{p_i}.$$

Now, we juxtapose to n the (natural) number

$$RF(n) = \prod_{i=1}^{k} p_i^{\alpha_i - 1}.$$

that we shall call *strong restrictive factor.*

It can be easily seen that if for every i $(1 \leq i \leq k)$ $\alpha_i = 1$, then $RF(n) = 1$.

On the other hand, if there is at least one $\alpha_i > 1$, then

$$n > RF(n) > 1.$$

It can be easily seen that RF is a multiplicative function and

$$RF(\prod_{i=1}^{k} p_i^{2\alpha_i}) = \prod_{i=1}^{k} p_i^{2\alpha_i - 1} > \prod_{i=1}^{k} p_i^{2\alpha_i - 2} = (\prod_{i=1}^{k} p_i^{\alpha_i - 1})^2 = (RF(\prod_{i=1}^{k} p_i^{\alpha_i}))^2.$$

Moreover, let $n = k.l$, $m = k.s$ for $(k,l) = (k,s) = (l,s) = 1$. Then

$$RF(m,n) = RF(k^2).RF(l).RF(s) > RF(k)^2.RF(l).RF(s) = RF(m).RF(n).$$

Also, $RF(n) = 1$ iff $n = \prod_{i=1}^{k} p_i$. In particular, $RF(p) = 1$ for each prime number p.

RF is not a monotonous function. Its first 40 values are the following

n	RF(n)	n	RF(n)	n	RF(n)	n	RF(n)
1	1	11	1	21	1	31	1
2	1	12	2	22	1	32	16
3	1	13	1	23	1	33	1
4	2	14	1	24	4	34	1
5	1	15	1	25	5	35	1
6	1	16	8	26	1	36	6
7	1	17	1	27	9	37	1
8	4	18	3	28	2	38	1
9	3	19	1	29	1	39	1
10	1	20	2	30	1	40	4

For each natural number n: $RF(n) > \varphi(n)$.

THEOREM 1: For every natural number n:

$$A(n) \equiv n^2 - \varphi(n).\sigma(n) - RF(n) \geq 0.$$

Proof: Let $\underline{dim}(n) = 1$, i.e., n is a prime number. Then

$$n^2 - \varphi(n).\sigma(n) - RF(n) = n^2 - (n-1).(n+1) - 1 = 0.$$

Let us suppose that for every natural number n, if $1 \leq \underline{dim}(n) \leq k$, then the assertion is valid. We shall prove that if conditions $\underline{dim}(n') = k+1$ are valid for the natural number n', then the assertion of the Theorem is valid for n', too. However, $n' = n.p$, where $\underline{dim}(n) = k$ and p is a prime number.

Two cases are possible for p:

First case: $p \notin \underline{set}(n)$. Then

$$A(n') = n^2p^2 - \varphi(n.p).\sigma(n.p) - RF(n).RF(p)$$

$$= n^2p^2 - \varphi(n).\sigma(n)(p^2-1) - RF(n)$$

$$> p^2.(n^2 - \varphi(n).\sigma(n) - RF(n)) \geq 0.$$

Second case: $p \in \underline{set}(n)$. Then $n' = m.p^a$, where $\underline{dim}(m) \leq k-1,\ a \geq 1$ and

$$A(n') = m^2p^{2a+2} - \varphi(m.p^{a+1}).\sigma(m.p^{a+1}) - RF(m).RF(p^{a+1})$$

$$= m^2p^{2a+2} - \varphi(m).\sigma(m).p^a.(p^{a+2}-1) - RF(m).RF(p^{a+1})$$

$$> p^{2a+2}(m^2 - \varphi(m).\sigma(m) - RF(m))\ geq0.$$

Therefore,

$$n^2 - \varphi(n).\sigma(n) \geq RF(n).$$

Analogously it can be proved that

$$\varphi(n) + \sigma(n) - 2n \geq RF(n).$$

REFERENCES:

[1] Atanassov K., Irrational factor: definition, properties and problems, Notes on Number Theory and Discrete Mathematics, Vol. 2, 1996, No. 3, 42-44.

[2] Atanassov K., Converse factor: definition, properties and problems, Notes on Number Theory and Discrete Mathematics, Vol. 8, 2002, No. 1, 37-38.