NNTDM 8 (2002) 2, 75-76

MODIFICATION OF WEIERSTRASS'S INEQUALITY

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The following Weierstrass's inequality is well-known:

$$\prod_{i=1}^{n}(1+a_i) \geq 1+\sum_{i=1}^{n} a_i, \tag{1}$$

where $a_1, a_2, ..., a_n$ are positive real numbers (see, e.g., [1]).

Let for each positive real number a: $[a]$ denotes its integer part and let $\{a\} = a - [a]$.

We shall modify Weierstrass's inequality to the form (for the same numbers):

$$\prod_{i=1}^{n}(1+a_i) \geq \prod_{i=1}^{n}(1+[a_i]) + \sum_{i=1}^{n}\{a_i\}. \tag{2}$$

The proof is trivial, e.g., by induction. Really, for $n = 1$, i.e., for only one positive real number a_1, (2) it obvious. If we assume that it is valid for some n real numbers $a_1, a_2, ..., a_n$, then by induction:

$$\prod_{i=1}^{n+1}(1+a_i) \geq (\prod_{i=1}^{n}(1+a_i) + \sum_{i=1}^{n}\{a_i\})(1+[a_{n+1}]+\{a_{n+1}\})$$

$$= \prod_{i=1}^{n+1}(1+[a_i]) + (1+[a_{n+1}]).\sum_{i=1}^{n}\{a_i\} + \{a_{n+1}\}.(\prod_{i=1}^{n}(1+[a_i]) + \sum_{i=1}^{n}\{a_i\})$$

$$\geq \prod_{i=1}^{n+1}(1+[a_i]) + \sum_{i=1}^{n}\{a_i\} + \{a_{n+1}\} \geq \prod_{i=1}^{n+1}(1+[a_i]) + \sum_{i=1}^{n+1}\{a_i\}.$$

Now, we shall note that

$$\prod_{i=1}^{n}(1+[a_i]) + \sum_{i=1}^{n}\{a_i\} - 1 - \sum_{i=1}^{n} a_i = \prod_{i=1}^{n}(1+[a_i]) - 1 - \sum_{i=1}^{n}[a_i] \geq 0,$$

because (1) for $[a_1], [a_2], ..., [a_n]$. Therefore, (2) is more powerful than (1).

REFERENCE:

[1] Eames, W. The Elementary Theory of Numbers, Polynomials, and Rational Functions. Oldbourne, London, 1967.