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CONVERSE FACTOR: DEFINITION, PROPERTIES AND PROBLEMS

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The paper is a continuation of [1], where the concept of irrational factor has been introduced.

Every natural number n has a canonical representation in the form $n = \prod_{i=1}^{k} p_i^{\alpha_i}$, where $p_1, p_2, ..., p_k$ are different prime numbers and $\alpha_1, \alpha_2, ..., \alpha_k \geq 1$ are natural numbers.

In [1] it is juxtaposed to n the (real) number $IF(n) = \prod_{i=1}^{k} p_i^{1/\alpha_i}$.

Now, we juxtapose to n the (natural) number $n' = \prod_{i=1}^{k} \alpha_i^{p_i}$.

It can be easily seen that if for every i $(1 \leq i \leq k)$ $\alpha_i = 1$, then $n' = 1$. On the other hand, if there is at least one $\alpha_i > 1$, then $n' \geq 1$.

Let us denote n' by $CF(n)$ and let us name it "Converse Factor" of n.

It can be seen that CF is a multiplicative function.

Indeed, let m and n be two natural numbers, for which $(m, n) = 1$. Therefore, if n has the above form and $m = \prod_{j=1}^{l} q_j^{\beta_j}$, where $q_1, q_2, ..., q_j$ are different prime numbers, for every i $(1 \leq i \leq k)$ and for every j $(1 \leq j \leq l) : p_i \neq q_j$, and $\beta_1, \beta_2, ..., \beta_l \geq 1$ are natural numbers, then

$$CF(n.m) = \prod_{i=1}^{k} \alpha_i^{p_i} . \prod_{j=1}^{l} \beta_j^{q_j} = CF(n).CF(m).$$

On the other hand, if for the prime numbers a, b, c: $m = a.b$ and $n = b.c$, then

$$CF(m.n) = CF(a.b^2.c) = 2^b > 1 = CF(ab).CF(bc) = CF(m).CF(n).$$

CF is not a monotonous function.

If $k = l, p_1 = q_1, ..., p_k = q_l$ and $\alpha_1 \geq \beta_1, \alpha_2 \geq \beta_2, ...\alpha_k \geq \beta_l$, then $n \geq m$ and $CF(n) \geq CF(m)$. The inequality will be strong if at least one of the inequalities between α_i and β_i is strong $(1 \leq i \leq k)$.

If $k < l, p_1 = q_1, ..., p_k = q_k$ and $\alpha_1 = \beta_1, \alpha_2 = \beta_2, ...\alpha_k = \beta_k$, then $n < m$ and $CF(n) \leq CF(m)$.

There is no relation between n and m when $k \neq l$.

Obviously, $CF(n) \geq 1$ for every natural number $n > 1$; and for every two natural numbers n and m:

$$CF(n^m) = CF(\prod_{i=1}^{k} p_i^{m\alpha_i}) = \prod_{i=1}^{k} (m\alpha_i)^{p_i} = m^{\sum_{i=1}^{k} \alpha_i}.CF(n).$$

In [2] is defined the following function, too: $\zeta(n) = \sum_{i=1}^{k} \alpha_i p_i$. For every natural number n:

$$\zeta(n) = \sum_{i=1}^{k} \alpha_i p_i = \sum_{i=1}^{k} p_i \alpha_i = \zeta(CF(n)).$$

For the Möbius function μ (see e.g., [3]) for every natural number n is valid the following equality

$$\mu(n) = (-1)^{\underline{cas}(n)+1}.[\frac{1}{CF(n)}], \qquad (1)$$

where for every integer number n function $\underline{cas}(n)$ is the number of the prime divisors of n and $[x]$ is the integer part of real number $x \geq 0$.

Really, if n is a prime number, then

$$(-1)^2[\frac{1}{1}] = 1 = \mu(n);$$

if $n = p_1p_2...p_s$, where $p_1, p_2, ..., p_s$ are prime numbers, then $\underline{cas}(n) = s$ and

$$(-1)^{s+1}[\frac{1}{CF(p_1p_2...p_s)}] = (-1)^{s+1}[\frac{1}{1}] = (-1)^{s+1} = \mu(n);$$

if there exists such a prime p that $n = p^s m$, where s, m are natural numbers, m does not divide by p and $s \geq 2$, then

$$(-1)^{\underline{cas}(n)+1}[\frac{1}{CF(p^s m)}] = (-1)^{\underline{cas}(n)+1}[\frac{1}{s^p.CF(m)}] = 0 = \mu(n),$$

because at least $s^p > 1$.

It can be easily seen that

$$IF(n) = n \quad \text{if and only if} \quad CF(n) = 1 \quad \text{if and only if} \quad |\mu(n)| = 1.$$

REFERENCES:

[1] Atanassov K., Irrational factor: definition, properties and problems, Notes on Number Theory and Discrete Mathematics, Vol. 2, 1996, No. 3, 42-44.

[2] Atanassov K., New integer functions, related to φ and σ functions, Bull. of Number Theory and Related Topics Vol. XI (1987), No. 1, 3-26.

[3] Chandrasekharan K. Introduction to analytic number theory, Springer-Verlag, Berlin, 1968.