NNTDM 6 (2000) 2, 61-63

CONNECTIONS IN MATHEMATICS: FIBONACCI SEQUENCE VIA ARITHMETIC PROGRESSION

Krassimir T. Atanassov

CLBME - Bulg. Academy of Sci., P.O.Box 12, Sofia-1113, /ulgaria,

e-mail: krat@bgcict.acad.bg

The idea for this short remark was generated by the Marchisotto's paper [1]. Thus I borrow the first part of its title and offer to all colleagues to prepare a series of papers under the first part of this title.

Here we shall discuss an approach for an interpretation of the Fibonacci sequence as an arithmetic progression. The reasoning for this is the fact, that there is a relation between the way of generating the Fibonacci sequence and the way of generating the arithmetic progression. On the other hand, obviously, the Fibonacci sequence is not an ordinary arithmetic progression. Thus we can construct a new type of progression which will include both the ordinary arithmetic progression, and the Fibonacci sequences (the classical one and its generalizations).

Let $f : \mathcal{N} \to \mathcal{R}$ be a fixed function, where $\mathcal{N}$ and $\mathcal{R}$ are the sets of the natural and real numbers, respectively, and a be a fixed real number. The sequence

$$a,\ a+f(1),\ a+f(2),\ ...,\ a+f(k),\ ... \qquad (*)$$

we shall call A-progression (from "arithmetic progression").

Obviously, if $a_k = a + f(k)$ is its $k-$th member, then

$$\sum_{k=0}^{n} a_k = (n+1).a + \sum_{k=1}^{n} f(k).$$

When $f(k) = k.d$ for the fixed real number d we obtain from (*) the ordinary arithmetic progression.

When $a = 0$ and f is the function defined by:

$$f(1) = 1,\ f(2) = 1,\ f(k+2) = f(k+1) + f(k) \ \text{ for } k \geq 1,$$

we obtain from (*) the ordinary Fibonacci sequence: 0, 1, 1, 2,...

Therefore, the ordinary Fibonacci sequence can be represented by an A-progression. We shall show that some of the generalizations of this sequence can be represented by an A-progression, too. When a and b are fixed real numbers and f is a function defined by

$$f(1) = b - a,\ f(2) = b,\ f(k+2) = f(k+1) + f(k) + a,$$

we obtain from (*) the generalized Fibonacci sequence

$$a, b,\ a + b,\ a + 2.b,\ 2.a + 3.b, \ldots$$

(see e.g. [2]). When a, b and c are fixed real numbers and f is a function defined by

$$f(1) = b - a,\ f(2) = c - a,\ f(3) = b + c,$$

$$f(k+3) = f(k+2) + f(k+1) + f(k) + 2.a,$$

we obtain from (*) the generalized Fibonacci sequence named Tribonacci sequence (see e.g. [2]):

$$a,\ b,\ c,\ a + b + c,\ a + 2.b + 2.c,\ 2.a + 3.b + 4.c, \ldots$$

When a, b, c and d are fixed real numbers, and f and g are functions defined by:

$$f(1) = -a + b,\ f(2) = -a + c + d,$$

$$f(k+2) = g(k+1) + g(k) - a + 2.c\ (k \geq 1)$$

$$g(1) = -c + d,\ g(2) = a + b - c,$$

$$g(k+2) = f(k+1) + f(k) + 2.a - c\ (k \geq 1)$$

we obtain from (*) the generalization of the Fibonacci sequence from [3]. When for the same a, b, c and d

$$f(1) = -a + b,\ f(2) = -a + b + c,$$

$$f(k+2) = f(k+1) + g(k) + c\ (k \geq 1)$$

$$g(1) = -c + d,\ g(2) = a - c + d,$$

$$g(k+2) = g(k+1) + f(k) + a\ (k \geq 1)$$

we obtain from (*) the generalization of the Fibonacci sequence from [4-6].

REFERENCES:

[1] Marchisotto E., Connections in mathematics: as introduction to Fibonacci via Pythagoras. The Fibonacci Quarterly, Vol. 31 (1993), No. 1, 21-27.

[2] Vorob'ev N. Fibonacci numbers, Pergamon Press, London, 1961.

[3] Atanassov K., Atanassov L., Sasselov D. A new perspective to the generalization of the Fibonacci sequence, The Fibonacci Quartarly Vol. 23 (1985), No. 1, 21-28.

[4] Atanassov K., On a second new generalization of the Fibonacci sequence. The Fibonacci Quarterly Vol. 24 (1986), No. 4, 362-365.

[5] Lee J.-Z., Lee J.-S., Some properties of the generalization of the Fibonacci sequence. The Fibonacci Quarterly, Vol. 25 (1987), No. 2, 111-117.

[6] Spickerman W., Joyner R., Creech R., On the (2, F) generalizations of the Fibonacci sequence, The Fibonacci Quartarly, Vol. 30 (1992), No. 4, 310-314.