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ON THE 125-th SMARANDACHE'S PROBLEM

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The following Smarandache's problem is formulated in [1]:
To prove that

$$n! > k^{n-k+1} \prod_{i=0}^{k-1} [\frac{n-i}{k}]! \qquad (*)$$

for any non-null positive integers n *and* k.

Below we shall introduce a solution of this problem.

Let everywhere k be a fixed natural number. Obviously, if for some n: $k > n$, then the inequality (*) is obvious, because its right side is equal to 0. Also obviously it can be seen that (*) is valid for $n = 1$. Let us assume that (*) is valid for some natural number n. Then

$$(n+1)! - k^{n-k+2} \prod_{i=0}^{k-1} [\frac{n-i+1}{k}]!$$

(by the induction assumption)

$$> (n+1).k^{n-k+1} \prod_{i=0}^{k-1} [\frac{n-i}{k}]! - k^{n-k+2} \prod_{i=0}^{k-1} [\frac{n-i+1}{k}]!$$

$$k^{n-k+1} \prod_{i=1}^{k-1} [\frac{n-i}{k}]!.((n+1).[\frac{n-k+1}{k}]! - k.[\frac{n+1}{k}]! \geq 0,$$

because

$$(n+1).[\frac{n-k+1}{k}]! - k.[\frac{n+1}{k}]! = (n+1).[\frac{n-k+1}{k}]! - k.[\frac{n-k+1}{k}+1]!$$

$$= [\frac{n-k+1}{k}]!.(n+1-k.[\frac{n+1}{k}]) \geq 0.$$

With this the problem is solved.

REFERENCES:

[1] Dumitrescu C., Seleacu V., Problem 125. Some Notions and Questions in Number Theory. Erhus Univ. Press, Glendale, 1994.