

FIVE SMARANDACHE CONJECTURES ON PRIMES

edited by M.L.Perez

American Research Press

Rehoboth, Box 141, NM 87322, USA,

E-mail: m_l_perez@yahoo.com

1) The equation

$$p_{n+1}^x - p_n^x = 1,$$

where p_n is the n -th prime, has a unique solution between 0.5 and 1;

- the maximum solution occurs for $n = 1$, i.e.

$$3^x - 2^x = 1 \text{ when } x = 1;$$

- the minimum solution occurs for $n = 31$, i.e.

$$127^x - 113^x = 1 \text{ when } x = 0.567148... = a_0.$$

Thus, Andrica's conjecture

$$A_n = p_{n+1}^{1/2} - p_n^{1/2} < 1$$

is generalized to:

2) $B_n = p_{n+1}^a - p_n^a < 1$ where $a < a_0$.

3) $C_n = p_{n+1}^{1/k} - p_n^{1/k} < 2/k$ where $k \geq 2$.

4) $D_n = p_{n+1}^a - p_n^a < 1/n$ where $a < a_0$ and n big enough, $n = n(a)$, holds for infinitely many consecutive primes.

a) Is this still available for $a_0 < a < 1$?

b) Is there any rank n_0 depending on a and n such that (4) is verified for all $n \geq n_0$?

5) $p_{n+1}/p_n \leq 5/3$, and the maximum occurs at $n = 2$.

References:

- [1] Sloane, N.J.A., Sequence A001223/M0296 (“Andrica’s Conjecture”) in <An On-Line Version of the Encyclopedia of Integer Sequences>.
- [2] Sloane, N.J.A., Sequence A038458 (“Smarandache Constant” = 0.56714813020201771464684687553348256458679024938863820684028522182...) in <An On-Line Version of the Encyclopedia of Integer Sequences>, <http://www.research.att.com/cgi-bin/access.cgi/as/njas/sequences/eishis.cgi>.
- [3] Smarandache, Florentin, “Conjectures which Generalize Andrica’s Conjecture”, Arizona State University, Hayden Library, Special Collections, Tempe, AZ, USA.
- [4] Weisstein, Eric W., “CRC Concise Encyclopedia of Mathematics”, CRC Press, Boca Raton, FL, USA, p. 44, 1998.