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ON THE 118-th SMARANDACHE'S PROBLEM

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The following Smarandache's problem is formulated in [1] with the title "Smarandache criterion for coprimes":

If a, b are strictly positive integers, then: a and b are coprimes if and only if

$$a^{\varphi(b)+1} + b^{\varphi(a)+1} \equiv a + b (\mathrm{mod}\ ab),$$

where φ is Euler's totient.

Below we shall introduce a solution of one direction of this problem and we shall introduce a counterexample to the other direction of the problem.

Let a, b are strictly positive integers for which $(a, b) = 1$. Hence, from one of the Euler's theorems (see, e.g., [2]) it follows that

$$a^{\varphi(b)} \equiv 1 (\mathrm{mod}\ b)$$

and

$$b^{\varphi(b)} \equiv 1 (\mathrm{mod}\ a).$$

Therefore

$$a^{\varphi(b)+1} \equiv a (\mathrm{mod}\ ab)$$

and

$$b^{\varphi(b)+1} \equiv b (\mathrm{mod}\ ab)$$

from where it follows that really

$$a^{\varphi(b)+1} + b^{\varphi(a)+1} \equiv a + b (\mathrm{mod}\ ab).$$

It can be seen easily that the other direction of the Smarandashe's problem is not valid. For example, if $a = 6$ and $b = 10$, and therefore $(a, b) = 2$, then:

$$6^{\varphi(10)+1} + 10^{\varphi(6)+1} = 6^5 + 10^3 = 7776 + 1000 = 8776 \equiv 16 (\mathrm{mod}\ 60).$$

Therefore, the "Smarandache criterion for coprimes" is not correct.

REFERENCES:

[1] Dumitrescu C., Seleacu V., Problem 118. Some Notions and Questions in Number Theory. Erhus Univ. Press, Glendale, 1994.

[2] Nagell T., Introduction to Number Theory. John Wiley & Sons, Inc., New York, 1950.