NNTDM 5 (1999) 3, 97-98

ON THE 117-th SMARANDACHE'S PROBLEM

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The following Smarandache's problem is formulated in [1]:
Let p be an odd positive number. Then p and p + 2 are twin primes if and only if

$$(p-1)!(\frac{1}{p}+\frac{2}{p+2})+\frac{1}{p}+\frac{1}{p+2}$$

is an integer.

Below we shall introduce a solution of this problem.

Let

$$A \equiv (p-1)!(\frac{1}{p}+\frac{2}{p+2})+\frac{1}{p}+\frac{1}{p+2} = \frac{(p-1)!(3p+2)+2p+2}{p(p+2)} = \frac{B}{p(p+2)},$$

where

$$B \equiv (p-1)!(3p+2)+2p+2.$$

Hence

$$B = 3p! + 2p + 2((p-1)!+1).$$

Therefore, $p|B$ iff $p|((p-1)!+1)$ iff p is a prime number (from Wilson's theorem - see, e.g. [2]).

On the other hand

$$B = (p+2)(p-1)! + 2(p+2) + 2p! - 2$$

$$= (p+2)(p-1)! + 2(p+2) + \frac{2}{p+1}((p+1)! - (p+1))$$

$$= (p+2)(p-1)! + 2(p+2) + \frac{2}{p+1}(((p+1)!+1) - (p+2)).$$

Therefore (from $(p+1, p+2) = 1$ for $p \geq 2$), $(p+2)|B$ iff $(p+2)|((p+1)!+1)$ iff $p+2$ is a prime number (from Wilson's theorem).

Hence, $p(p+2)|B$ iff p and $p+2$ are twin primes. Therefore, A is an integer iff p and $p+2$ are twin primes. With this we solved the problem.

Finally, we shall note that in [3] the following assertion is proved:

p and p + 2 are twin primes iff $p(p+2)|C$, *where*

$$C = 4(p-1)! + p + 4.$$

It is easily to see that

$$B = C + 3p(2(p-1)! + 1). \qquad (*)$$

From $(p+2)|(2(p-1)!+1)$ iff $(p+2)$ is a prime number, from (*) and from the above assertion from [3] we obtain another proof of the Smarandache's problem. Also, our first proof and (*) yields another proof of the assertion from [3].

REFERENCES:

[1] Dumitrescu C., Seleacu V., Problem 117. Some Notions and Questions in Number Theory. Erhus Univ. Press, Glendale, 1994.

[2] Nagell T., Introduction to Number Theory. John Wiley & Sons, Inc., New York, 1950.

[3] Ribenboim R., The book of Prime Number Records, Springer-Verlag, New York, 1989.