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ON THE 97-th, THE 98-th AND THE 99-th SMARANDACHE'S PROBLEMS

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The 97-th problem from [1] is the following (see also Problem 80 from [2]):

Smarandache constructive set (of digits 1,2):

$$1, 2, 11, 12, 21, 22, 111, 112, 121, 122, 211, 212, 221, 222, 1111, 1112, 1121, 1122,$$

$$1211, 1212, 1221, 1222, 2111, 2112, 2121, 2122, 2211, 2212, 2221, 2222, ...$$

(Numbers formed by digits 1 and 2 only.)

Definition:
a1) 1, 2 belongs to S_2;
a2) if a, b belongs to S_2, then $\overline{ab}$ belongs to S_2 too;
a3) only elements obtained by rules a1) and a2) applied a finite number of times belong to S_2.

Remark:
- there are 2^k numbers of k digits in the sequence, for $k = 1, 2, 3, ...$;
- to obtain from the $k-$digits number group the $(k+1)-$digits number group, just put first the digit 1 and second the digit 2 in the front of all $k-$digits numbers.

The 98-th problem from [1] is the following (see also Problem 81 from [2]):

Smarandache constructive set (of digits 1,2,3):

$$1, 2, 3, 11, 12, 13, 21, 22, 23, 31, 32, 33, 111, 112, 113, 121, 122, 123, 131, 132, 133, 211,$$

$$212, 213, 221, 222, 223, 231, 232, 233, 311, 312, 313, 321, 322, 323, 331, 332, 333, ...$$

(Numbers formed by digits 1, 2, and 3 only.)

Definition:
a1) 1, 2, 3 belongs to S_3;
a2) if a, b belongs to S_3, then $\overline{ab}$ belongs to S_3 too;
a3) only elements obtained by rules a1) and a2) applied a finite number of times belong to S_3.

Remark:
- *there are 3^k numbers of k digits in the sequence, for $k = 1, 2, 3, \ldots$;*
- *to obtain from the $k-$digits number group the $(k+1)-$digits number group, just put first the digit 1, second the digit 2, and third the digit 3 in the front of all $k-$digits numbers.*

The 99-th problem from [1] is the following (see also Problem 82 from [2]):

Smarandache generalized constructive set:
(Numbers formed by digits $d_1, d_2, \ldots, d_m$ only, and d_i being different each other, $1 \leq m \leq 9$.)

Definition:
a1) $d_1, d_2, \ldots, d_m$ belongs to S_m;
a2) if a, b belongs to S_2, then $\overline{ab}$ belongs to S_2 too;
a3) only elements obtained by rules a1) and a2) applied a finite number of times belong to S_m.

Remark:
- *there are m^k numbers of k digits in the sequence, for $k = 1, 2, 3, \ldots$;*
- *to obtain from the $k-$digits number group the $(k+1)-$digits number group, just put first the digit d_1, second the digit d_2, ..., and the $m-$time digit d_m in the front of all $k-$digits numbers.*

More general: all digits d_i can be replaced by numbers as large as we want (therefore of many digits each), and also m can be as large as we want.

As in the previous sections, we can construct new sequences for every one of the three sequences in the following forms, respectively:

$1, 2,$
$11, 12, 21, 22,$
$111, 112, 121, 122, 211, 212, 221, 222,$
$1111, 1112, 1121, 1122, 1211, 1212, 1221, 1222, 2111, 2112, 2121, 2122, 2211, 2212, 2221, 2222, \ldots$

$1, 2, 3,$
$11, 12, 13, 21, 22, 23, 31, 32, 33,$
$111, 112, 113, 121, 122, 123, 131, 132, 133, 211, 212, 213, 221, 222, 223, 231, 232, 233,$
$311, 312, 313, 321, 322, 323, 331, 332, 333,$
$1111, 1112, 1113, \ldots$

$$d_1, d_2, \ldots, d_m$$
$$d_1d_1, d_1d_2, \ldots, d_1d_m, d_2d_1, \ldots d_md_m,$$
$$d_1d_1d_1, d_1d_1d_2, \ldots, d_md_md_m, \ldots$$

As it is noted in the begining of the section, the number of the members of the $k-$th row in the first, second and third sequence in the new form will be respectively 2^k, 3^k and m^k.

Let us mark the three sequences, respectively by S_2, S_3 and S_m. Therefore, we can represent these sets, respectively, by:

$$S_2 = \bigcup_{n=1}^{\infty} \{\overline{a_1 a_2 \ldots a_n} \mid a_1, a_2, \ldots, a_n \in \{1, 2\}\} \equiv \bigcup_{n=1}^{\infty} A_{2,n}$$

and, as it was mentioned above,

$$card(A_{2,n}) = 2^n,$$

where $card(X)$ is the cardinality of the set X;

$$S_3 = \bigcup_{n=1}^{\infty} \{\overline{a_1 a_2 \ldots a_n} \mid a_1, a_2, \ldots, a_n \in \{1, 2, 3\}\} \equiv \bigcup_{n=1}^{\infty} A_{3,n}$$

and

$$card(A_{3,n}) = 3^n;$$

$$S_m = \bigcup_{n=1}^{\infty} \{\overline{a_1 a_2 \ldots a_n} \mid a_1, a_2, \ldots, a_n \in \{d_1, d_2, \ldots, d_m\}\} \equiv \bigcup_{n=1}^{\infty} A_{m,n}$$

and

$$card(A_{m,n}) = m^n.$$

In the general (third) case we shall define:

$$B_{m,n} = \sum_{x \in A_{m,n}} x.$$

Therefore

$$B_{2,1} = 3 = 2^0.3.1,$$
$$B_{2,2} = 66 = 2^1.3.11,$$
$$B_{2,3} = 1332 = 2^2.3.111,$$
$$B_{2,4} = 26664 = 2^3.3.1111, \ldots$$

$$B_{3,1} = 6 = 3^0.6.1,$$
$$B_{3,2} = 198 = 3^1.6.11,$$
$$B_{3,3} = 5994 = 3^2.6.111,$$
$$B_{3,4} = 59994 = 3^3.6.1111, \ldots$$

It is interesting to note, for example, that

$$B_{4,1} = 4^0.10.1.$$

$$B_{4,2} = 440 = 4^1.10.11,$$
$$B_{4,3} = 17760 = 4^2.10.111, \ldots$$

Now we can prove by induction that

$$B_{m,n} = m^{n-1}.(\sum_{i=1}^{m} d_i).\underbrace{11\ldots1}_{n\text{times}}. \qquad (1)$$

Really, for m - fixed natural number and $n = 1$ we obtain that

$$B_{m,1} = \sum_{i=1}^{m} d_i = m^0.(\sum_{i=1}^{m} d_i).1.$$

Let us assume that $B_{m,n}$ satisfies (1) for some natural number $n \geq 1$ (m is fixed). Then from the above construction it is seen that

$$B_{m,n+1} = m.(m^{n-1}.(\sum_{i=1}^{m} d_i).\underbrace{11\ldots1}_{n\text{times}}) + m^n.10^n.(\sum_{i=1}^{m} d_i)$$

$$= m^n.(\sum_{i=1}^{m} d_i).(1\underbrace{00\ldots0}_{n\text{times}} + \underbrace{11\ldots1}_{n\text{times}})$$

$$= m^{(n+1)-1}.(\sum_{i=1}^{m} d_i).\underbrace{11\ldots1}_{(n+1)\text{times}}),$$

with which (1) is proved.

Below using the usual notation $[x]$ for the integer part of the real number x, we shall give a formula for the s-th member $x_{m,s}$ of the general (third) sequence. The validity of this formula is proved also by induction. It is:

$$x_{m,s} = \sum_{i=1}^{[log_m(s+1)(m-1)]} 10^{i-1}.(r([\frac{s - m.[\frac{m^{i-1}-1}{m-1}]}{m^{i-1}}], m) + 1), \qquad (2)$$

where

$$r(p,q) = p - q.[\frac{p}{q}]$$

for every two natural numbers p and q, i.e., function r determines the remainder of the division of p by q.

When $m = 2$, (2) obtained the form

$$x_{2,s} = \sum_{i=1}^{[log_2(s+1)]} 10^{i-1}.(r([\frac{s - 2.m^i + m}{m^{i-1}}], 2) + 1),$$

and when $m = e$, (2) obtained the form

$$x_{m,s} = \sum_{i=1}^{[log_3 2.(s+1)]} 10^{i-1}.(r([\frac{s - 3.[\frac{m^{i-1}-1}{2}]}{3^{i-1}}], 3) + 1).$$

Using formula (2) we can show the s–th partial sum of the third sequence (and from there - of the first and the second sequences). It is

$$S_{m,s} = \sum_{i=1}^{s} x_{m,s},$$

but we can construct the following simpler formula from a calculating point of view, having in mind that the s–th member of the third sequence is placed in the $([log_m((s-2)(m-1)+1)] + 1)$–th subsequence and also the sume of the members of the first $([log_m((s-2)(m-1)+1)]$ sequences can be calculated by (1):

$$S_{m,s} = \sum_{i=1}^{[log_m((s-2)(m-1)+1)]} B_{m,i} + \sum_{i=s-t+1}^{s} x_{m,s},$$

where

$$t = \frac{m^{[log_m((s-2)(m-1)+1)]} - 1}{m-1}.$$

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[1] C. Dumitrescu, V. Seleacu, Some notions and questions in number theory, Erhus Univ. Press, Glendale, 1994.

[2] F. Smarandache, Only problems, not solutions!. Xiquan Publ. House, Chicago, 1993.