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ON THE 37-th AND 38-th SMARANDACHE'S PROBLEMS

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The 37-th and 38-th problems from [1] are the following (see also Problem 39 from [2]):
(Inferior) prime part:

$$2, 3, 3, 5, 5, 7, 7, 7, 7, 11, 11, 13, 13, 13, 13, 17, 17, 19, 19, 19, 19, 23, 23, 23, 23, 23, 23, 29, 29,$$

$$31, 31, 31, 31, 31, 31, 37, 37, 37, 37, 41, 41, 43, 43, 43, 43, 47, 47, 47, 47, 47, 47, 53, 53, 53,$$

$$53, 53, 53, 59, \ldots$$

(For any positive real number n one defines $p_p(n)$ as the largest prime number less than or equal to n.)
(Superior) prime part:

$$2, 2, 2, 3, 5, 5, 7, 7, 11, 11, 11, 11, 13, 13, 17, 17, 17, 17, 19, 19, 23, 23, 23, 23, 29, 29, 29,$$

$$29, 29, 29, 31, 31, 37, 37, 37, 37, 37, 37, 41, 41, 41, 41, 43, 43, 47, 47, 47, 47, 53, 53, 53,$$

$$53, 53, 53, 59, 59, 59, 59, \ldots$$

(For any positive real number n one defines $P_p(n)$ as the smallest prime number greater than or equal to n.)
Study these sequences.

First, we must note that in the first sequence $n \geq 2$, while in the second one $n \geq 0$. It is better, if the first two members of the second sequence are omitted. Let everywhere below $n \geq 2$.

Second, let us denote by $\{p_1, p_2, p_3, \ldots\} = \{2, 3, 5, \ldots\}$, the set of all prime numbers. Let $p_0 = 1$, and let $\pi(n)$ be the number of the prime numbers less or equal to n (see e.g., [3]).

Then the n-th member of the first sequence is $p_p(n) = p_{\pi(n)-1}$ and of the second sequence is $P_p(n) = p_{\pi(n)+\mathcal{B}(n)}$, where

$$\mathcal{B}(n) = \begin{cases} 0, & \text{if } n \text{ is a prime number} \\ 1, & \text{otherwise} \end{cases}$$

(see [4]).

The checks of these equalities are straightforward, or by the induction.

Therefore, the values of the n-th partial sums

$$X_n = \sum_{k=1}^{n} p_p(k)$$

and

$$Y_n = \sum_{k=1}^{n} P_p(k)$$

of the two Smarandache's sequences are, respectively,

$$X_n = \sum_{k=2}^{\pi(n)} (p_k - p_{k-1}).p_{k-1} + (n - p_{\pi(n)} + 1).p_{\pi(n)} \tag{1}$$

and

$$Y_n = \sum_{k=1}^{\pi(n)} (p_k - p_{k-1}).p_k + (n - p_{\pi(n)}).p_{\pi(n)+\mathcal{B}(n)}. \tag{2}$$

The proofs can be made by the induction. For example, the validity of (2) is proved as follows.

Let n = 2. Then the validity of (2) is obvious. Let us assume that (2) is valid for some natural number n. For the forms of n and $n+1$ there are three cases:
(a) n and $n+1$ are not prime numbers. Therefore, $\pi(n+1) = \pi(n)$ and $\mathcal{B}(n+1) = \mathcal{B}(n) = 1$, and then

$$X_{n+1} = Y_n + P_p(n+1)$$

$$= \sum_{k=1}^{\pi(n)} (p_k - p_{k-1}).p_k + (n - p_{\pi(n)}).p_{\pi(n)+\mathcal{B}(n)} + p_{\pi(n+1)+\mathcal{B}(n+1)}$$

$$= \sum_{k=1}^{\pi(n+1)} (p_k - p_{k-1}).p_k + (n - p_{\pi(n+1)}).p_{\pi(n+1)+\mathcal{B}(n+1)} + p_{\pi(n+1)+\mathcal{B}(n+1)}$$

$$= \sum_{k=1}^{\pi(n+1)} (p_k - p_{k-1}).p_k + ((n+1) - p_{\pi(n+1)}).p_{\pi(n+1)+\mathcal{B}(n+1)}.$$

(b) n is a prime number. Therefore, for $n > 2$ $n+1$ is not a prime number, $\pi(n+1) = \pi(n)$, $n = p_{\pi(n)}$, $\mathcal{B}(n) = 0$, $\mathcal{B}(n+1) = 1$, and then

$$Y_{n+1} = Y_n + P_p(n+1)$$

$$= \sum_{k=1}^{\pi(n)} (p_k - p_{k-1}).p_k + (n - p_{\pi(n)}).p_{\pi(n)+\mathcal{B}(n)} + p_{\pi(n+1)+\mathcal{B}(n+1)}$$

(from $n - p_{\pi(n)} = 0$ and $n + 1 - p_{\pi(n+1)} = n + 1 - p_{\pi(n)} = 1$)

$$= \sum_{k=1}^{\pi(n+1)} (p_k - p_{k-1}).p_k + ((n+1) - p_{\pi(n+1)}).p_{\pi(n+1)+\mathcal{B}(n+1)}.$$

(c) $n+1$ is a prime number. Therefore, for $n > 2$ n is not a prime number, $\pi(n+1) = \pi(n)+1$, $n+1 = p_{\pi(n+1)}$, $\mathcal{B}(n) = 1$, $\mathcal{B}(n+1) = 0$, and then

$$Y_{n+1} = Y_n + P_p(n+1)$$

$$= \sum_{k=1}^{\pi(n)} (p_k - p_{k-1}).p_k + (n - p_{\pi(n)}).p_{\pi(n)+\mathcal{B}(n)} + p_{\pi(n+1)+\mathcal{B}(n+1)}$$

(from $p_{\pi(n)+\mathcal{B}(n+1)} = p_{\pi(n)+1+0} = p_{\pi(n)+\mathcal{B}(n)}$)

$$= \sum_{k=1}^{\pi(n)} (p_k - p_{k-1}).p_k + ((n+1) - p_{\pi(n)}).p_{\pi(n)+\mathcal{B}(n)}$$

$$= \sum_{k=1}^{\pi(n)} (p_k - p_{k-1}).p_k + (p_{\pi(n+1)} - p_{\pi(n)}).p_{\pi(n+1)}$$

$$= \sum_{k=1}^{\pi(n+1)} (p_k - p_{k-1}).p_k + ((n+1) - p_{\pi(n+1)}).p_{\pi(n+1)+\mathcal{B}(n+1)}$$

Therefore, (2) is valid.

The validity of (1) is proved analogically.

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