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ON THE 4-th SMARANDACHE'S PROBLEM

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The 4-th problem from [1] (see also 18-th problem from [2]) is the following:

Smarandache deconstructive sequence:

$$\underbrace{1,\ 23,\ 456,\ 789}\underbrace{1,\ 23456,\ 789}\underbrace{123,\ 456789}\underbrace{1,\ 23456789},$$
$$\underbrace{123456789},\ \underbrace{123456789}1,\ \ldots$$

Let the n-th term of the above sequence is a_n. Then we can see that the first digits of the first nine members are, respective: 1, 2, 4, 7, 2, 7, 4, 2, 1. Let us define the function ω as follows:

r	0	1	2	3	4	5	6	7	8	9
$\omega(r)$	1	1	2	4	7	2	7	4	2	1

Here we shall use the arithmetic function ψ, discussed shortly in §14 and detailed in the author's paper [3] .

Now, we can prove that the form of the $n-$th member of the above sequence is

$$a_n = \overline{b_1 b_2 ... b_n},$$

where $b_1 = \omega(n - [\frac{n}{9}])$, $b_2 = \psi(\omega(n - [\frac{n}{9}]) + 1)$, ..., $b_n = \psi(\omega(n - [\frac{n}{9}]) + n - 1)$.

Every natural number n can be represented in the form

$$n = 9q + r,$$

where $q \geq 0$ is a natural number and $r \in \{1, 2, ..., 9\}$.

We shall prove by induction that the forms of nine sequentially members a_{n+1}, $a_{n+2}, \ldots$, a_{n+9}, where $n = 9q + r$, are the following:

$$a_{9q+1} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{q \text{ times}}1$$

$$a_{9q+2} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{q \text{ times}}23$$

$$a_{9q+3} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{q \text{ times}}456$$

$$a_{9q+4} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{q \text{ times}}7891$$

$$a_{9q+5} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{q \text{ times}}23456$$

$$a_{9q+6} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{q \text{ times}}789123$$

$$a_{9q+7} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{q \text{ times}}4567891$$

$$a_{9q+8} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{q \text{ times}}23456789$$

$$a_{9q+9} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{q+1 \text{ times}}$$

When $q = 0$ the validity of the above assertion is obvious. Let us assume that for some natural number q, a_{9q+1}, a_{9q+2}, ... a_{9q+9} have the above forms. Then for a_{9q+10}, a_{9q+11}, ... a_{9q+18} we obtain the following representations, putting $p = q + 1$:

$$a_{9q+10} = a_{9p+1} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{p \text{ times}}1$$

$$a_{9q+11} = a_{9p+2} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{p \text{ times}}23$$

$$a_{9q+12} = a_{9p+3} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{p \text{ times}}456$$

$$a_{9q+13} = a_{9p+4} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{p \text{ times}}7891$$

$$a_{9q+14} = a_{9p+5} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{p \text{ times}}23456$$

$$a_{9q+15} = a_{9p+6} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{p \text{ times}}789123$$

$$a_{9q+16} = a_{9p+7} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{p \text{ times}}4567891$$

$$a_{9q+17} = a_{9p+8} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{p \text{ times}}23456789$$

$$a_{9q+18} = a_{9p+9} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{p+1 \text{ times}}$$

On the above sequence $\{a_n\}_{n=1}^{\infty}$ we can juxtapose the sequence $\{\psi(a_n)\}_{n=1}^{\infty}$ for which we can prove (as above) that it has the basis $[1, 5, 6, 7, 2, 3, 4, 8, 9]$.

The problem can be generalized, e.g., to the following for:

Study the sequence $\{a_n\}_{n=1}^{\infty}$, *which* $s-th$ *member has the form*

$$a_s = \overline{b_1 b_2 ... b_{s.k}},$$

where $b_1 b_2 ... b_{s.k} \in \{1, 2, ..., 9\}$ *and* $b_1 = \omega'(s - [\frac{s}{9}])$, $b_2 = \psi(\omega'(s - [\frac{s}{9}]) + 1)$, ..., $b_{s.k} = \psi(\omega'(s - [\frac{s}{9}]) + s.k - 1)$, *and here*

r	1	2	3	4	5	6
$\omega'(r)$	1	$\psi(k+1)$	$\psi(3k+1)$	$\psi(6k+1)$	$\psi(10k+1)$	$\psi(15k+1)$

r	7	8	9
$\omega'(r)$	$\psi(21k+1)$	$\psi(28k+1)$	$\psi(36k+1)$

For example, when $k = 2$:

$$\underbrace{12,\ 3456,\ 789}\underbrace{123,\ 456789}\underbrace{12,\ 3456789}\underbrace{123,\ 456789}\underbrace{123456,}$$

$$\underbrace{789}\underbrace{123456789}\underbrace{12,\ 3456789}\underbrace{123456789,\ 123456789}\underbrace{123456789},...$$

On the last sequence $\{a_n\}_{n=1}^{\infty}$ *we can juxtapose again the sequence* $\{\psi(a_n)\}_{n=1}^{\infty}$ *for which we can prove (as above) that it has the basis* $[3, 9, 3, 6, 3, 6, 9, 8, 9]$.

REFERENCES:

[1] *C. Dumitrescu, V. Seleacu, Some Sotions and Questions in Number Theory, Erhus Univ. Press, Glendale, 1994.*

[2] *F. Smarandache, Only Problems, Not Solutions!. Xiquan Publ. House, Chicago, 1993.*