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CIRCUMFERENCES WITH GOLDEN POINTS

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Devoted to Prof. John Turner
for his 70-th birthday

Up to now triangles and squares with golden points on their sides have been defined in [1] and some of their properties been examined.

In that paper a new geometrical object is suggested - a Circumference with Golden Points (CGP).

Let us have a segment of an arbitrary length c and its golden points (see Fig. 1).

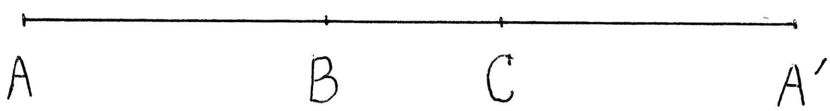

Fig. 1

Let us connect both of the ends of the segment so as to have a circumference with three points - A, B and C (see Fig. 2).

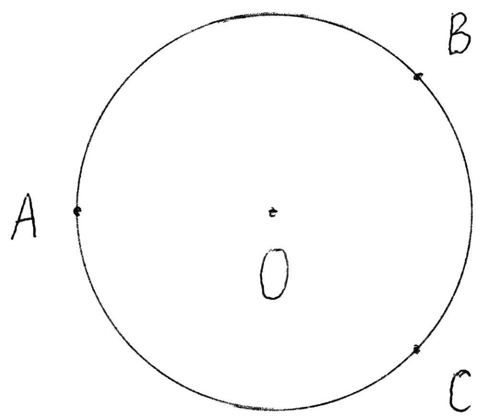

Fig. 2

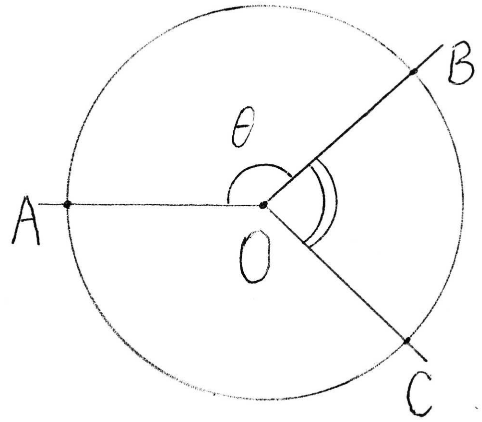

Fig. 3

So, we define a CGP as a circumference with a point - called "initial" (A) and two points (B and C) called "golden" if and only if the proportion between the lengths of the catenaries

$$\frac{\overset{\frown}{AB}}{\overset{\frown}{BC}} = \frac{\overset{\frown}{ACB}}{\overset{\frown}{AB}} = \frac{c}{\overset{\frown}{ACB}},$$

is valid.

Knowing that the length of the catenary is measured by means of the respective central angle (see Fig. 3):

$$\frac{\theta}{2(\pi - \theta)} = \frac{2\pi - \theta}{\theta} = \frac{2\pi}{2\pi - \theta}.$$

Let us prove that the golden points of any circumference with the same centre lie on the two straight lines OB and OC (see Fig. 4).

From the proportion we see that

$$\frac{\overset{\frown}{AB}}{\overset{\frown}{BC}} = \frac{\overset{\frown}{A_1B_1}}{\overset{\frown}{B_1C_1}} = \frac{\theta}{2(\pi - \theta)},$$

and when A_1, A and O lie on a straight line, their left and right golden points will also lie on two straight lines - left and right.

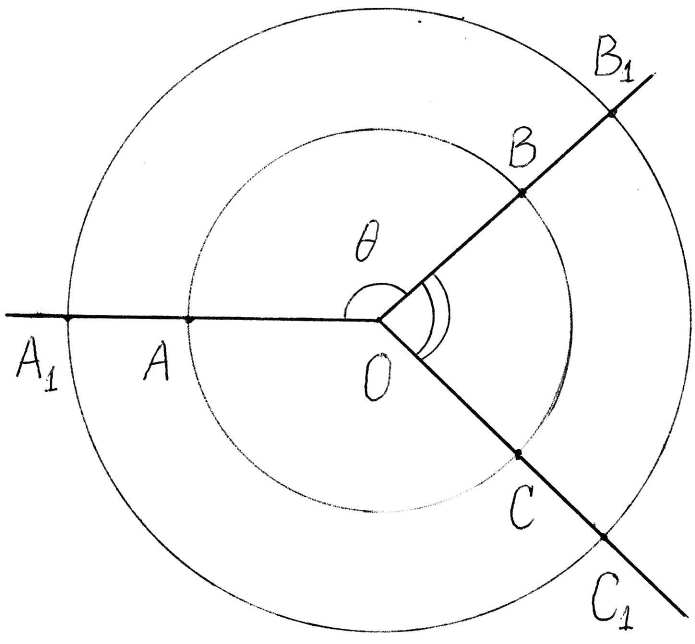

Fig. 4

Thus we proved the universality of that property of the newly found CGP.

Now, let us prove another similar property. Let us have two circumferences, the smaller of which is inscribed in the other, as shown on Fig. 5; AA' being the diameter of the inner circumference K, A_1A' - the diameter of the outer circumference, and have A_1, A and A' lying on the same straight line.

Let us fix the golden points on the inner circumference and connect A' with B and C. Let the common point of the line $A'A$ and K_1 be A_1, of the line $A'B$ and K_1 be B_1, and of the line $A'C$ and K_1 be C_1.

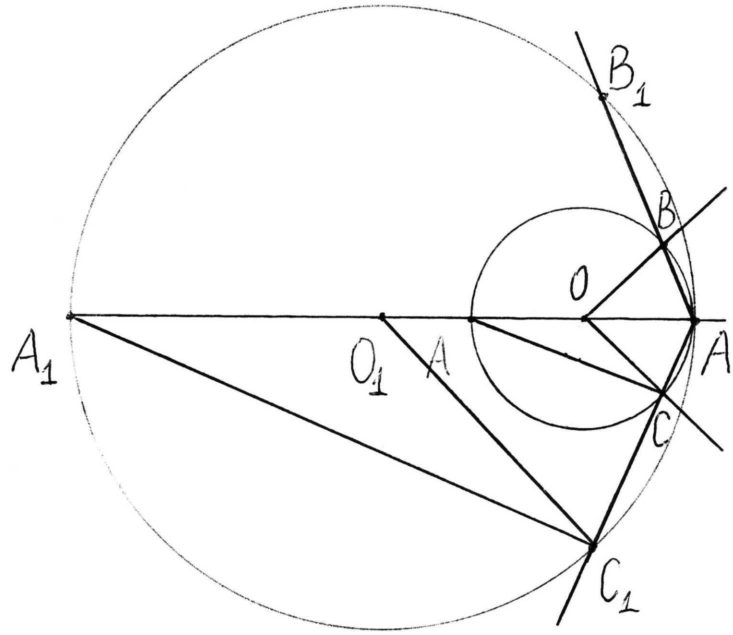

Fig. 5

Let us connect AC and A_1C_1 thus we have the right-angled triangles ACA' and A_1C_1A' which are similar, because of having two corresponding couples of equal angles $\angle AA'C = \angle A_1A'C_1$ and $\angle ACA' = \angle A_1C_1A'$. Hence, when their hypothenuses lie on the same straight line, their medians are parallel to each other and $\angle CAA' = \angle C_1A_1A'$ from where the parts, to which the catenaries $\widehat{A'C}$ and $\widehat{A'C_1}$ divide the circumferences, are equal and C_1 is a golden point of the bigger outer circumference.

REFERENCE:

[1] K. T. Atanassov, V. K. Atanassova, A. G. Shannon, J. C. Turner, Visual Perspectives on Number Sequences, In preparation, 1999.