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A MODIFICATION OF SIVARAMAKRISHNAN-VENKATARAMAN'S INEQUALITY

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Abstract

A modifications of one Sivaramakrishnan - Venkataraman's inequality is proposed and proved.

In [1] it is given the following inequality of R. Sivaramakrishnan and C. Venkataraman:

$$\sigma(n) \geq d(n).\sqrt{n}, \tag{1}$$

where for every natural number $n = \prod_{i=1}^{k} p_i^{\alpha_i}$, for which k, α_1, α_2, ... $\alpha_k \geq 1$ are natural numbers and $p_1, p_2, ..., p_k$ are different prime numbers:

$$\sigma(n) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1} - 1}{p_i - 1},$$

$$d(n) = \prod_{i=1}^{k} (\alpha_i + 1).$$

Here we shall prove that

$$\psi(n) \geq d(n).\sqrt{n}, \tag{2}$$

where

$$\psi(n) = \prod_{i=1}^{k} p_i^{\alpha_i - 1}.(p_i + 1).$$

Obviously, for every natural number n:

$$\sigma(n) \geq \psi(n).$$

Let for the above n:

$$set(n) = \{p_1, p_2, \ldots p_k\},$$

$$dim(n) = \sum_{i=1}^{k} \alpha_i.$$

For $dim(n) = 1$ (2) is valid, because in this case n is a prime number and

$$\psi(n) - d(n).\sqrt{n} = n + 1 - 2.\sqrt{n} \geq 0.$$

First, we shall prove that (2) is valid for every odd number.

Let us assume that (2) be valid for every n such that $s \geq dim(n) \geq 1$. Let n be a fixed natural number, such that $dim(n) = s$ and let p be a fixed prime number. For p there are two cases.

Case 1. $p \notin set(n)$. Then $dim(n.p) = dim(n) + 1$ and

$$\psi(n.p) - d(n.p).\sqrt{n.p} = \psi(n).(p+1) - 2.d(n).\sqrt{n.p}$$

$$\geq (p+1).d(n).\sqrt{n} - 2.d(n).\sqrt{n.p} = d(n).\sqrt{n}.(p + 1 - 2.\sqrt{n.p}) \geq 0.$$

Case 2. $p \in set(n)$. Then $dim(n.p) = dim(n)+1$, $n+p^a.m$ for some natural numbers $a, m \geq 1$, m satisfies (2), because $dim(m) \leq s$, and

$$\psi(n.p) - d(n.p).\sqrt{n.p} = \psi(n).p - d(m.p^{a+1}).\sqrt{n.p}$$

$$= \psi(n).p - d(m).(a+2).\sqrt{n.p} \geq p.d(n).\sqrt{n} - d(m).(a+2).\sqrt{n.p}$$

$$= d(m).\sqrt{n.p}((a+1).\sqrt{p} - (a+2)) \geq d(m).\sqrt{n.p}((a+1).\sqrt{3} - (a+2)) > 0.$$

Second, we shall prove that (2) is valid for every even number, too.

Let $n = 2.m$, where m is an odd number and let for m (2) be vaild. Then

$$\psi(2.m) - d(2.m).\sqrt{2.m} = 3.\psi(m) - 2.d(m).\sqrt{2.m}$$

$$\geq 3.d(m).\sqrt{m} - 2.d(m).\sqrt{2.m} = d(m).\sqrt{m}(3 - 2.\sqrt{2}) > 0.$$

Let $n = 2^a.m$, where $a \geq 2$ is a natural number, m is an odd number, and let for m (2) be vaild. Then

$$\psi(2^a.m) - d(2^a.m).\sqrt{2^a.m} = 3.2^{a-1}.\psi(m) - (a+1).d(m).\sqrt{2^a.m}$$

$$\geq 3.2^{a-1}.d(m).\sqrt{m} - (a+1).d(m).\sqrt{2^a.m} = d(m).\sqrt{2^a.m}(3.2^{\frac{a-2}{2}} - (a+1)) \geq 0.$$

The check of the last inequality for $a \geq 2$ is trivial. This completes the proof. ◊

REFERENCE:

[1] D. Mitrinovic, J. Sandor, Handbook of Number Theory, Kluwer Academic Publishers, 1996.