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THREE HYPOTHESES ON φ AND σ FUNCTIONS

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Three hypotheses concerning φ and σ functions will be discussed. The idea was generated in the last year, when the author showed that all numbers smaller than 1000 are representable only by the digits "1", "9", "9" and "4", operation "+" and functions φ and σ. Its results were planned for publication in "Bulletin on Number Theory", but the issue for 1994 was not printed in time and the paper somewhat lost in value. Now, we extend its idea and formulate some open problems related to it.

Let F, G and H be strings of the symbols "φ" and "σ", where for every natural number $n = \prod_{i=1}^{k} p_i^{\alpha_i}$ (where $k, \alpha_1, \alpha_2, \ldots, \alpha_k \geq 1$ are natural numbers and $p_1, p_2, \ldots, p_k$ are prime numbers):

$$\varphi(x) = \prod_{i=1}^{k} p_i^{\alpha_i - 1} . (p_i - 1) \text{ and } \sigma(n) = \prod_{i=1}^{k} (p_i^{\alpha_i + 1} - 1)/(p_i - 1).$$

For example, if F = $\sigma\varphi\sigma$, G = $\varphi\varphi\sigma\sigma$ and H = $\emptyset$ (the empty string) then:

$F(7) = \sigma(\varphi(\sigma(7))) = \sigma(\varphi(8)) = \sigma(4) = 7$

$G(7) = \varphi(\varphi(\sigma(\sigma(7)))) = \varphi(\varphi(\sigma(8))) = \varphi(\varphi(15)) = \varphi(8) = 4$

$H(7) = \emptyset(7) \equiv 7.$

The following HYPOTHESIS is interesting.

HYPOTHESIS 1: Let x, y and z be arbitrary natural numbers. Then there exist strings F, G and H for which:

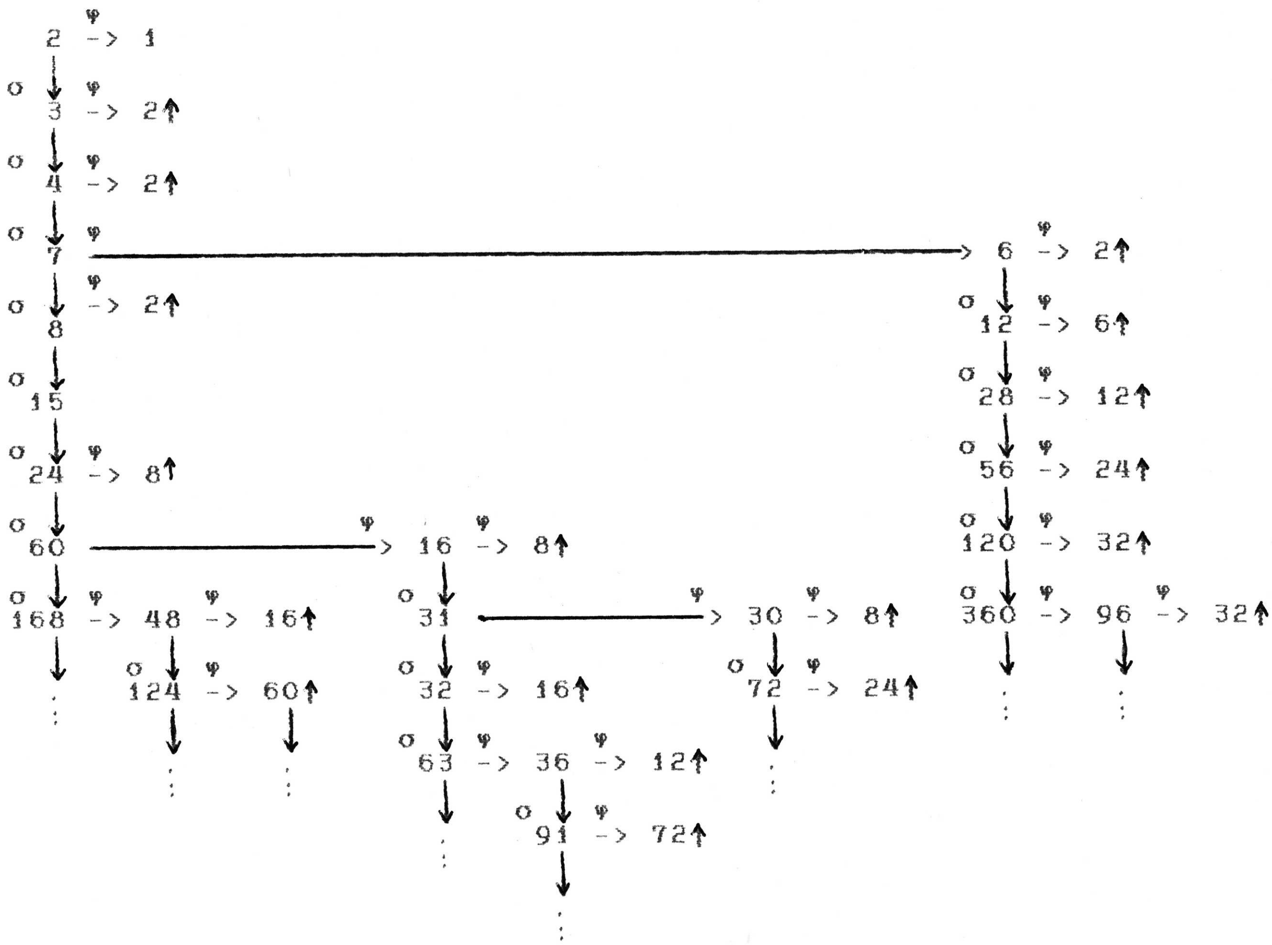
$z = F(G(x) + H(y)).$

A simpler but equivalent form of this hypothesis is the following

HYPOTHESIS 2: Let z be an arbitrary natural number. Then there exist strings G and H for which:

$z = G(2) + H(2).$

We can construct the following scheme, where the "$\uparrow$" symbol denotes that the corresponding number is already encountered in the scheme:



The following question is interesting.

Let the string F be called a Σ-string, if for some natural number x: F(x) = x. Let the Σ-string F be called a Σ_n-string, if F is a Σ-string and the number of its elements (symbols "φ" and "σ") is n, where n ≥ 2 is a natural number.

HYPOTHESIS 3: For every natural number n ≥ 2 there exists at least one Σ_n-string.

For example: φ(σ(6)) = 6, φ(σ(σ(2))) = 2, φ(σ(φ(σ(60)))) = 60, φ(φ(σ(σ(σ(8))))) = 8, etc.