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REMARKS ON PRIME NUMBERS. II

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Let $p_0 = 2$, $p_1 = 3$, $p_2 = 5$, ... be the sequence of the prime numbers. Let

$$\partial(x) = \begin{cases} 1, & \text{if } x \text{ is a prime number and it is the greater from prime twins} \\ 0, & \text{otherwise} \end{cases}$$

Here we shall prove the following assertion.

THEOREM: For every two natural numbers k, $n \geq 1$:

$$p_{n+k} - p_n \geq 2.k + \pi(n + k) - \pi(n) - \partial(n + k). \qquad (*)$$

Proof: Let everywhere n be a fixed natural number. Let $k = 1$. If $p_{n+1} = p_n + 2$, then $\partial(n + 1) = 1$ and

$$A \equiv p_{n+1} - p_n - 2 - \pi(n + 1) + \pi(n) + \partial(n + 1)$$

$$= - \pi(n + 1) + \pi(n) + 1 \geq 0.$$

If p_{n+1} is not the greater from prime twins, then $\partial(n + 1) = 0$ and

$$A \geq p_{n+1} - p_n - 2 - \pi(n + 1) + \pi(n)) \geq 4 - 2 - 1 > 0.$$

Let us assume that (*) is valid for some natural number $k \geq 1$. Let

$$A \equiv p_{n+k+1} - p_n - 2.(k + 1) - \pi(n + k + 1) + \pi(n) + \partial(n + k + 1).$$

If $p_{n+k+1} = p_{n+k} + 2$, then $\partial(n + k + 1) = 1$, $\partial(n + k) = 0$, $\pi(n + k + 1) \leq \pi(n + k) + 1$ and from (*):

$$A = p_{n+k} + 2 - p_n - 2.(k + 1) - \pi(n + k + 1) + \pi(n) + 1$$

$$\geq p_{n+k} - p_n - 2.k - \pi(n + k) + \pi(n) + \partial(n + k)) \geq 0.$$

If p_{n+k+1} is not the greater from prime twins, then $\partial(n + k + 1) = 0$, $\partial(n + k) \leq 1$, $\pi(n + k + 1) \leq \pi(n + k) + 1$ and from (*):

$$A \geq p_{n+k} + 4 - p_n - 2.(k + 1) - \pi(n + k + 1) + \pi(n)$$

$$\geq p_{n+k} - p_n - 2.k - \pi(n + k) + \pi(n) + \partial(n + k) \geq 0.$$

Therefore, the Theorem is valid. It can be formulated also in the form: for every two natural numbers $n \geq 2$ and m $(1 \leq m \leq n - 1)$:

$$p_n - p_m \geq 2.(n - m) + \pi(n) - \pi(m) - \partial(n).$$

Let for every natural number x:

$$\omega(x) = \begin{cases} 1, & \text{if } x \text{ is a prime number} \\ 0, & \text{otherwise} \end{cases}$$

Therefore, $\omega(x) \geq \partial(x)$. From it, for $m = 1$ we obtain (cf. [1]):

$$p_n \geq 2.n + 1 + \pi(n) - \partial(n) > 2.n + \pi(n) - \partial(n) - \partial(p_n - 2)$$

$$\geq 2.n + \pi(n) - \omega(n) - \omega(p_n - 2),$$

i.e., the Theorem 1 from [1] is proved.

REFERENCE:

[1] Atanassov K., Remarks on prime numbers, Bull. of Number Theory (in press).