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The structure of prime sums

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Abstract: The development of prime sequences has allowed the detailed structure of prime sums to be made. Such structures help to explain why some sums yield primes while others give composites. The application of right-end-digit (modulo 10) structure permits analysis independent of the size of the integers being examined.
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1 Introduction

Prime sums feature in a number of conjectures, such as that all odd integers greater than 5 can be formed from the sum of three primes [cf. 1]. Recent studies of prime sequences [2, 3, 4] allow the structure of the sums to be analysed so that the primality of the sum can be explained.

This is done by considering the integers in the form nR where R is the right-end-digit and n the remaining digits. For example, for $N = 20779$, $n = 2077$ and $R = 9$. The work is predicated on the only REDs for primes are 1, 3, 7, 9, though not every integer with a RED of 1, or 3, or 7, or 9, is necessarily prime.

The n values can be classified for our purposes into three sequences $\{3t\}$, $\{3t + 1\}$, $\{3t + 2\}$. Integers can never be prime when

- $n \in \{3t\}, R \in \{3, 9\}$, or
- $n \in \{3t + 2\}, R \in \{1, 7\}$.

Furthermore, an integer can also have an n value which prevents the formation of a prime for certain R values if n belongs to an imbedded sequence of the form $\{a + bj\}$, in which a has the form of the main sequence and $3|b$ (Table 1).

$R = 1$	(a, b)
$n \in \{3t\}$	(3, 93) [n>3], (9, 21), (9, 39), (12, 33), (36, 57), (39, 51), (84, 87)
$n \in \{3t + 1\}$	(1, 66) [n>1], (16, 21), (16, 69), (22, 39), (22, 51), (34, 66)
$n \in \{3t + 2\}$	No primes formed
$R = 3$	(a, b)
$n \in \{3t\}$	No primes formed
$n \in \{3t + 1\}$	(1, 39), (13, 21), (13, 57), (25, 33), (25, 69), (49, 51)
$n \in \{3t + 2\}$	(2, 69) [n>2], (14, 33), (14, 39), (20, 21), (32, 51), (32, 57)
$R = 7$	(a, b)
$n \in \{3t\}$	(0, 21), (18, 33), (18, 51), (24, 39), (24, 57), (66, 69)
$n \in \{3t + 1\}$	(1, 51) [n>1], (7, 21), (7, 66), (37, 39), (43, 57)
$n \in \{3t + 2\}$	No primes formed
$R = 9$	(a, b)
$n \in \{3t\}$	No primes formed
$n \in \{3t + 1\}$	(1.57) [n>1], (4, 21), (16, 39), (28, 51), (31, 33), (52, 69), (64, 231), (79, 102)
$n \in \{3t + 2\}$	(11, 21), (11, 51), (20, 33), (20, 57), (29, 39), (29, 69), (32, 42), (89, 93)

Table 1. Imbedded sequences which block prime formation for particular REDs

2 Some prime sums

The n value of the sum of the REDs depends on the number of primes being added and the value of the RED; for instance, the sum of 9, 9, 9 is 27 with $n = 2$ and $R = 7$, whereas the sum of seven REDs of 9 would yield an n of 6 and an R of 3. To illustrate the detailed structure we use three-prime sums and the various R sets which give a sum with a particular R (Table 2).

	Some prime triples' REDs whose sums yield the *R*						
$n_R \in$	$\{3t\}$ 0		$\{3t+1\}$ 1		$\{3t+2\}$ 2		***R* sum**
			1, 1, 9	1, 3, 7	3, 9, 9	7, 7, 7	1
	1, 1, 1		1, 3, 9	3, 3, 7	7, 7, 9		3
	1, 3, 3		1, 7, 9	3, 7, 7	9, 9, 9		7
	1, 1, 7	3, 3, 3	3, 9, 7	1, 9, 9			9

Table 2. REDs of some prime sums

The detailed structure of the sums of three primes, chosen at random, are listed in Tables 3, 4, 5, and 6. The reasons for the primality of the sums are shown in the last column. Primes are formed when the n value of the sum is compatible with the main sequence and/or the imbedded sequence. In these tables,

$$t_0 = \left\lfloor \frac{n}{3} \right\rfloor,$$

so that

$$n = 3t_0 + a,\ a = 0,1\ or\ 2.$$

Set	p_1	p_2	p_3	$n_1 \in$	$n_2 \in$	$n_3 \in$	t_1	t_2	t_3	**Σ*n* =**
1, 1, 9 (11)	41	71	79	$\{3t_1+1\}$	$\{3t_2+1\}$	$\{3t_3+1\}$	1	2	2	3((Σt) + 1) = 18
1, 1, 9 (11)	41	71	89	$\{3t_1+1\}$	$\{3t_2+1\}$	$\{3t_3+2\}$	1	2	2	3(Σt + 1) + 1 = 19
1, 1, 9 (11)	31	61	89	$\{3t_1\}$	$\{3t_2\}$	$\{3t_3+2\}$	1	2	2	3(Σt) + 2 = 17
1, 1, 9 (11)	61	131	179	$\{3t_1\}$	$\{3t+1\}$	$\{3t_3+2\}$	2	4	5	3((Σt) + 1) = 36
1, 3, 7 (11)	211	653	157	$\{3t_1\}$	$\{3t_2+2\}$	$\{3t_3\}$	7	21	5	3(Σt) + 2 = 101
1, 3, 7 (11)	252	283	257	$\{3t_1+1\}$	$\{3t_2+1\}$	$\{3t_3+1\}$	8	9	8	3((Σt) + 1) = 78

Table 3(a). Structure of Σ*n* in three prime sum with *R* = 1

Set, ΣSet	p_1	p_2	p_3	n_R	***n* = *n*** $n_R + \Sigma n$	***a***	t_0	**Comments**
1, 1, 9 (11)	41	71	79	1	19	1	6	191 (p); sequences compatible
1, 1, 9 (11)	41	71	89	1	20	2	6	201 (c); when *n* = 3*t* + 2, *R* = 1 never prime
1, 1, 9 (11)	31	61	89	1	18	0	6	181 (p); sequences compatible
1, 1, 9 (11)	61	131	179	1	37	1	12	371 (c); main sequence compatible, but imbedded sequence *n* = 16 + 21*j*, *j* = 1 prevents *R* = 1 forming a prime (Table 1)
1, 3, 7 (11)	211	653	157	1	102	0	34	1021 (p); sequences compatible
1, 3, 7 (11)	252	283	257	1	79	1	26	791 (c); main sequence compatible; imbedded sequence *n* = 16 + 21*j*, *j* = 3 prevents *R* = 1 forming a prime

Table 3(b). Structure of three prime sum with *R* = 1

Set	p_1	p_2	p_3	$n_1 \in$	$n_2 \in$	$n_3 \in$	t_1	t_2	t_3	**Σ*n* =**
3, 3, 7	523	1783	2857	$\{3t_1+1\}$	$\{3t_2+1\}$	$\{3t_3\}$	17	59	95	3(Σt) + 2 = 515
7, 7, 9	5197	2917	1609	$\{3t_1\}$	$\{3t_2\}$	$\{3t_3+1\}$	173	97	53	3(Σt) + 1 = 970
7, 7, 9	337	1217	2579	$\{3t_1\}$	$\{3t_2+1\}$	$\{3t_3+2\}$	11	40	85	3(Σt + 1) = 411
1, 1, 1	5981	641	2621	$\{3t_1+1\}$	$\{3t_2+1\}$	$\{3t_3+1\}$	199	21	87	3(Σt + 1) = 924
1, 1, 1	5981	601	2621	$\{3t_1+1\}$	$\{3t_2\}$	$\{3t_3+1\}$	199	20	87	3(Σt) + 2 = 920

Table 4(a). Structure of Σ*n* in three prime sum with *R* = 3

Set, ΣSet	p_1	p_2	p_3	n_R	$n = n_R + \Sigma n$	a	t_0	Comments
3, 3, 7 (11)	523	1783	2857	1	516	0	172	5163 (c); main sequence incompatible
7, 7, 9 (23)	5197	2917	1609	2	972	0	324	9723 (c); main sequence incompatible
7, 7, 9 (23)	337	1217	2579	2	413	2	137	4133 (p); main sequence compatible; imbedded sequence $n = 14 + 21j$, $j = 19$
1, 1, 1 (03)	5981	641	2621	0	924	0	308	9243 (c); main sequence incompatible
1, 1, 1 (03)	5981	601	2621	0	920	2	306	9203 (p); main sequence compatible; imbedded sequence $n = 17 + 21j$, $j = 43$

Table 4(b). Structure of three prime sum with $R = 3$

Set	p_1	p_2	p_3	$n_1 \in$	$n_2 \in$	$n_3 \in$	t_1	t_2	t_3	Σn =
9, 9, 9	2039	929	2969	$\{3t_1 + 2\}$	$\{3t_2 + 2\}$	$\{3t_3 + 2\}$	67	30	98	3((Σt) + 2) = 591
1, 3, 3	571	2143	5153	$\{3t_1\}$	$\{3t_2 + 1\}$	$\{3t_3 + 2\}$	19	71	171	3((Σt) + 1) = 786
1, 7, 9	3461	4027	1319	$\{3t_1 + 1\}$	$\{3t_2\}$	$\{3t_3 + 2\}$	115	134	43	3((Σt) + 1) = 879
7, 7, 3	4177	2897	2293	$\{3t_1\}$	$\{3t_2 + 1\}$	$\{3t_3 + 1\}$	139	96	76	3(Σt) + 2 = 935

Table 5(a). Structure of Σ*n* in three prime sum with $R = 7$

Set, ΣSet	p_1	p_2	p_3	n_R	n	a	t_0	Comments
9, 9, 9 (27)	2039	929	2969	2	593	2	197	5937 (c); Main sequence prevents primes as $R = 1$ or 7 yields a composite (Table 1)
1, 3, 3 (07)	571	2143	5153	0	786	0	262	7867 (p); main sequence compatible; and imbedded sequence $n = 9 + 21j$, $j = 37$
1, 7, 9 (17)	3461	4027	1319	1	880	1	880	8807 (p); main sequence compatible; and imbedded sequence $n = 19 + 21j$, $j = 41$
7, 7, 3 (17)	4177	2897	2293	1	936	0	936	9367 (c); main sequence compatible; im-bedded sequence $n = 18 + 51j$, $j = 18$ (in Table 1)

Table 5(b). Structure of three prime sum with $R = 7$

Set, ΣSet	p_1	p_2	p_3	$n_1 \in$	$n_2 \in$	$n_3 \in$	t_1	t_2	t_3	**Σn =**
1, 1, 7	3701	5281	5987	$\{3t_1+1\}$	$\{3t_2\}$	$\{3t_3+1\}$	123	176	199	3(Σt) + 2 = 1496
9, 9, 1	166	97	130	$\{3t_1+1\}$	$\{3t_2+2\}$	$\{3t_3+1\}$	166	97	130	3((Σt) + 1) + 1 = 1183
3, 9, 7	3203	5209	5987	$\{3t_1+2\}$	$\{3t_2+1\}$	$\{3t_3+1\}$	106	173	199	3((Σt) + 1) + 1 = 1438
3, 3, 3	7393	6043	4243	$\{3t_1+1\}$	$\{3t_2+1\}$	$\{3t_3+1\}$	246	201	141	3((Σt) + 1) = 1767
3, 3, 3	1783	2843	3643	$\{3t_1+1\}$	$\{3t_1+2\}$	$\{3t_3+1\}$	59	94	121	3((Σt) + 1) + 1 = 826

Table 6(a). Structure of Σn in three prime sum with $R = 9$

Set, ΣSet	p_1	p_2	p_3	n_R	n	a	t_0	Comments
1, 1, 7 (09)	3701	5281	5987	0	1496	2	498	14969 (p); sequences compatible
9, 9, 1 (19)	166	97	130	1	1184	2	394	11849 (c); main sequence compatible; but not imbedded sequence $n = 11 + 51j$, $j = 23$ (Table 1)
3, 9, 7 (19)	3203	5209	5987	1	1439	2	479	14399(c); main sequence compatible; but not imbedded sequence $n = 11 + 21j$, $j = 68$ (Table 1)
3, 3, 3 (09)	7393	6043	4243	0	1767	0	589	17679 (c); main sequence blocks any primes for $R = 9$ (Table 1)
3, 3, 3 (09)	1783	2843	3643	0	826	1	275	8269 (p); sequences compatible

Table 6(b). Structure of three prime sum with $R = 9$

3 Concluding comments

3.1 Numerical results

A variety of sums for different numbers of arbitrary primes is listed in Tables 7(a) and 7(b):

No. of p	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	$\sum p$	$t_0 = \left\lfloor \frac{n}{3} \right\rfloor$
3	5227	3989	3121							12337	411
	2887	3701	5981							12569	418
	107	487	937							1531	51
5	5869	677	1193	1747	5153					14639	487
	5987	4283	2917	433	683					14303	476
7	2039	6229	7159	8053	6959	9349	641			40429	1347
	3623	4231	4877	5749	547	1447	1303			21777	725
9	5569	6217	2643	7901	8581	9697	5927	3313	3659	53507	1783
	2953	3371	3833	4363	5197	5827	5861	971	2297	32989	1099
	2953	3371	3833	4363	5197	5827	5861	971	2297	34673	1155

Table 7(a). Some prime sums

$\sum p$	p or c	Main sequence for $n = 3t_0 + a$	Imbedded sequence for n =	Comments
12337	c	$a = 0$	$24 + 39j$	$j = 31$ only composites when $R = 7$ for this sequence (Table 1)
12569	p	$a = 2$	$17 + 21j$	$j = 59$ (imbedded sequence not in Table 1)
1531	p	$a = 0$	$6 + 21j$	$j = 7$(imbedded sequence not in Table 1)
14639	p	$a = 2$	$14 + 21j$	$j = 69$ (imbedded sequence not in Table 1)
14303	p	$a = 2$	$2 + 21j$	$j = 68$ (imbedded sequence not in Table 1)
40429	p	$a = 1$	$10 + 21j$	$j = 192$ (imbedded sequence not in Table 1)
21777	c	$a = 2$		Main sequence incompatible – invalid $f(t)$ for $R = 7$, cannot be prime
53507	p	$a = 1$	$16 + 21j$	$j = 254$ (imbedded sequence not in Table 1)
32989	c	$a = 1$	$64 + 231j$	$j = 14$; only composites when $R = 9$ for this sequence of n (Table 1)
34673	p	$a = 2$	$2 + 21j$	$j = 165$; sequences compatible (not in Table 1)

Table 7(b). Comments for some prime sums

3.2 General results

Table 1 will obviously be incomplete as larger and larger integers are considered, and new imbedded sequences are revealed. However, the advantage of knowing the structural characteristics of the prime sums remains the same. The principles apply irrespective of the numbers of primes which are summed or the size of the integers as illustrated in Tables 7 (a) and (b).

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