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Alwyn Horadam: The man and his mathematics

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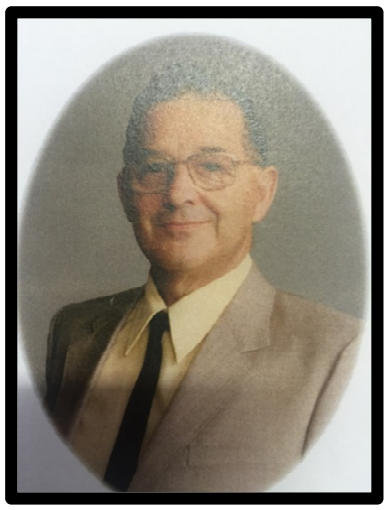
"Horrie"



Alwyn Francis ("Horrie") Horadam, was born 22 March 1923, son of a dairy farmer in the Hunter Valley about 230 km north of Sydney (Australia), and died 22 July 2016 in Armidale, a university town in the New England tablelands, 485 km north west of Sydney. He was educated at Maitland High School to which he had been awarded a scholarship, though this required about 120 km of travel each day during the depression. Despite the tiring effects of daily travel and working on the family farm he finished up as Dux of the School, Captain of the School, Captain of the Cricket team and a member of the school rugby team.

His athletic prowess in his younger days made it all the harder to witness his physical decline and confinement in his last years, partly the result of some post-operative problems following what should have been a simple procedure. Mentally though, he maintained his alertness with much reading from his collection of books, which grew in his nursing home room faster than he gave them away!

He had impeccable manners: courtesy personified! Though somewhat shy, other mathematicians enjoyed his company. As a research colleague and former student I found him to be positive, encouraging and constructive, with an eye for detail which he kept to the end – both mathematical detail and English grammar: no floating participles or split infinitives! His culture was epitomised at his funeral with passages from Shakespeare and Tennyson, as well reflective music by Vaughan Williams, Puccini and Massenet.

Alwyn had been at the University of New England as staff member or student almost continuously since its inception in 1938 as New England University College – the rural arm of the University of Sydney, Australia's oldest university. He graduated from that university with a *Bachelor of Arts (First Class Honours)*, *Diploma in Education and Doctor of Philosophy* (under Professor T. G. Room whose own "mathematical genealogy" included Henry Baker of Cambridge and Arthur Cayley of Oxford). He also completed a *Bachelor of Education* degree

with the University of Melbourne. His *Ph.D.* thesis was in projective geometry and involved work with Clifford matrices and showed the wide range of algebraic techniques, which he was later able to apply in number theory. Some of the publications which arose from this include [1, 2, 3, 4, 5, 6].

During his tenure at UNE, Alwyn filled many senior roles, including Head of the Mathematics Department, Dean of the Faculty of Science, and Member of the University Governing Council, as well as President of the University Union and Foundation Secretary of the UNE Teachers' Association, and senior executive and playing roles with the University Cricket and Football Clubs.

The Teacher

Appropriately for such a gifted teacher, on his grave will appear the line from the prologue of Geoffrey Chaucer's *The Canterbury Tales* in the original fourteenth century Middle English: *'And gladly wolde he lerne, and gladly teche'*. He literally taught thousands of students, very many of whom became high school mathematics teachers. He was a member of the government's secondary schools mathematics syllabus committee for more than twenty-five years, and he also co-authored a number of high school textbooks such as [7].

With undergraduates he pursued the ideal of educating in and through mathematics: a humane and liberal education. His teaching style was clear, precise and pedantic in that everything was written clearly on the board, and from memory because his preparation was so detailed and proper. He was always happy to answer questions, and his office door was actually always open. Two of his undergraduate textbooks were dealing with graph theory, block designs, and enumeration techniques, including recurrence relations and generating functions [8], as well as combinatorial structures and techniques [9, 10]. The most comprehensive of these texts [11] drew high praise from the eminent Oxford mathematician, W. L. Ferrar: *'What a task! – and how well it has been carried out. The task? A unified treatment of the Algebra, Geometry and Calculus considered basic for the foundation of undergraduate mathematics… Throughout, the author chooses every opportunity to interweave the variety of topics he is handling… The range of knowledge and detailed reference displayed by the author is most striking… That the author is an experienced teacher is everywhere apparent; he knows all the pitfalls'* [12].

The Mathematician

With his research and with his research degree students, of whom he had about fifty over many years, he encouraged a positive optimism and shrew guessing with correspondence to mathematicians around the world to avoid a parochial frame of mind wedded to the intellectual fashion of the day. He himself took advantage of the then generous sabbatical leave entitlements available to Australian academics and he spent periods of research at the Universities of North Carolina, Cambridge, Leeds, Liverpool, East Anglia, Reading, York, Exeter, Iceland and Malaya (where he was seconded to advise on the reform of their mathematics curriculum). He was on the Editorial Board of this journal and *The Fibonacci Quarterly* and regularly attended and presented papers at the biennial International Conferences on Fibonacci Numbers and Their

Applications, beginning with the initial conference at Patras in 1984. Subsequently he and Gerry Bergum and Andreas Philippou co-edited many volumes of the proceedings of these events, such as the one at Pisa [13].

While reference has already been made to Alwyn's interests in geometry, algebra and combinatorics, his most lasting legacy is likely to be in number theory where he generalized the Fibonacci sequence in the initial terms [14] and the recurrence relation [15]. The introduction of neat notation by considering the sequence

$$\{W_n\} \equiv \{W_n(a, b; p, q)\}$$

with a, b, p, q arbitrary integers, and defined by the second-order homogeneous linear recurrence relation

$$W_n = pW_{n-1} - qW_{n-2},\ n \geq 2$$

with initial conditions $W_0 = a$, $W_1 = b$; familiar now but novel then. Thus the sequence of ordinary Fibonacci numbers can be represented in this notation as $\{W_n(0, 1; 1, -1)\}$.

These generalizations were not only elegant, but they paved the way for clarifying the roles of the fundamental and primordial sequences introduced more than eighty years previously by Lucas [16]. They stimulated work on other second and higher order recursive sequences, and the introduction of many techniques from the special functions of mathematical physics [17].

Conclusion

In a short article like this, space does not permit comments at any great critical depth. It is hoped though that enough has been said to indicate that Horrie was a mathematician of international repute, a gentleman in the best sense of the word, industrious and well-rounded, loyal to people and to causes, who gave much to his profession and community – in fact a role model for an academic.

Given how busy he always he was it is almost a surprise to learn that he had time to get married in 1950 to Eleanor Mollie Horadam who died in 2002. Mollie was also a widely published mathematician of international repute, a graduate of London, Cambridge and New England Universities. They had three daughters (Kathie, Kerry and Alanna), five grand-daughters and one grand-son. Though both Mollie and Horrie were number theorists, they co-authored only one mathematics paper [18]. Parts of this tribute were taken from [19], which was written at the request of the then Editor of *The Fibonacci Quarterly* when Horrie retired thirty years ago (and checked by Mollie at the time).

Alwyn Francis Horadam – *vale*!

References

[1] Horadam, A.F. (1957) A locus in [8] invariant under a group of order 51840x81. *Quarterly Journal of Mathematics* (Oxford), Second Series 8: 241–259.

[2] Horadam, A.F. (1958) Projection of an invariant locus in [8] from a solid lying on it. *Quarterly Journal of Mathematics* (Oxford), Second Series 9: 81–86.

[3] Horadam, A.F. (1959) Involutions associated with the Burkhardt configuration in |4|. *Canadian Journal of Mathematics*. 11: 18–33.

[4] Horadam, A.F. (1959) Clifford groups in the plane. *Quarterly Journal of Mathematics* (Oxford), Second Series 10: 294–295.

[5] Horadam, A.F. (1961) Clifford matrices and the Hessian group. *Rendiconti del Circolo Matematico di Palermo*. Serie II, 10: 347–352.

[6] Horadam, A.F. (1966) Groups associated with certain loci in [5]. *Canadian Journal of Mathematics*. 18: 113–119.

[7] Horadam, A.F., M.E. Dunkley, I.W. Stewart. (1970) *New Horizons in Mathematics*. Sydney: Angus and Robertson.

[8] Horadam, A.F. (1979) Applied Combinatorics. Armidale: University of New England.

[9] Horadam, A.F. (1979) Combinatorial Mathematics. Armidale: University of New England.

[10] Horadam, A.F. (1970) *A Guide to Undergraduate Projective Geometry*. Sydney: Pergamon.

[11] Horadam, A.F. (1968) *Outline Course of Pure Mathematics*. Oxford: Pergamon.

[12] Ferrar, W.L. (1969) Review of Outline Course of Pure Mathematics by A.F. Horadam. *Mathematical Gazette*. 53: 448–449.

[13] Bergum, G.E., A.N. Philippou, A.F. Horadam (eds). (1989) *Applications of Fibonacci Numbers*, Volume 3. Dordrecht: Kluwer.

[14] Horadam, A.F. (1961) A generalized Fibonacci sequence. *American Mathematical Monthly*. 68: 455–459.

[15] Horadam, A.F. (1965) Generating functions for powers of a certain generalized sequence of numbers. *Duke Mathematical Journal.* 32: 437–446.

[16] Lucas, E. (1878) Théorie des fonctions numériques simplement périodiques. *American Journal of Mathematics*. 1: 184-240; 289–321.

[17] Horadam, A.F. (1977) Polynomials associated with Chebyshev polynomials of the first kind. *The Fibonacci Quarterly*. 15: 255–257.

[18] Horadam, A.F., E.M. Horadam. (1982) Roots of recurrence-generated polynomials. *The Fibonacci Quarterly*. 20: 219–226.

[19] Shannon, A.G. (1987) A.F. Horadam – ad multos annos. *The Fibonacci Quarterly*. 25: 100–104.