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Short remark on intuitionistic fuzziness and square-free numbers

Krassimir T. Atanassov

Department of Bioinformatics and Mathematical Modelling
Institute of Biophysics and Biomedical Engineering – Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 105, Sofia-1113, Bulgaria
e-mail: krat@bas.bg

Abstract: Two examples of possible relationships between elements of number theory and intuitionistic fuzzy set theory are given.
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In this short remark, we discuss an example of connection between the concepts of square-free natural numbers (see, e.g, [4]) and intuitionistic fuzziness (see, e.g. [2]). First, we give some definitions.

Let us have the natural number $n \geq 2$ with canonical representation

$$n = \prod_{i=1}^{k} p_i^{\alpha_i},$$

where k, α_1, ... $\alpha_k \geq 1$ are natural numbers and $p_1, ..., p_k$ are different prime numbers. Let us define for n the following well-known functions (cf., e.g, [4]):

$$\varphi(n) = \prod_{i=1}^{k} p_i^{\alpha_i - 1}.(p_i - 1),\ \varphi(1) = 1,$$

$$\psi(n) = \prod_{i=1}^{k} p_i^{\alpha_i - 1}.(p_i + 1),\ \psi(1) = 1,$$

$$\sigma(n) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i + 1} - 1}{p_i - 1},\ \sigma(1) = 1,$$

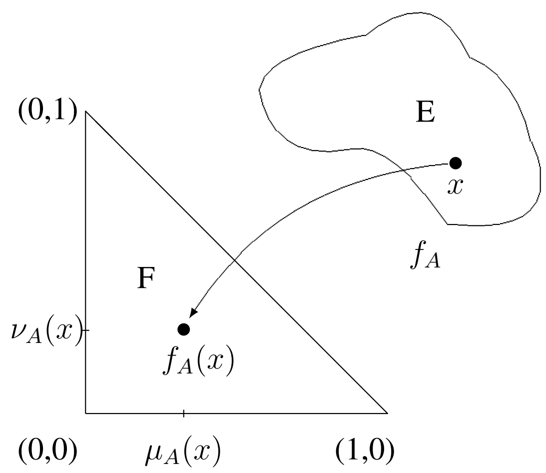
and (see [1])

$$\chi(n) = \prod_{i=1}^{k} (p_i^{\alpha_i} - p_i^{\alpha_i - 1} + ... + (-1)^{\alpha_i - 1} p_i + (-1)^{\alpha_i}).$$

The natural number n is square-free, if for any prime divisor p of n, p^2 does not divide n. For the above definitions, we see directly that for every square-free number n: $\psi(n) = \sigma(n)$, while for the non-square-free number n: $\sigma(n) > \psi(n)$.

The Intuitionistic Fuzzy Pair (IFP) is an object with the form $\langle a, b\rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$ [3].

Each IFP x corresponding to pair $\langle a, b\rangle$, can be represented as a point in the intuitionistic fuzzy interpretational triangle



Now, we introduce (for the first time) the concept of Fuzzy Pair (FP). The IFP will be called a FP if an only if $a = 1 - b$.

Let a set E be fixed. The Intuitionistic Fuzzy Set (IFS, see [2]) A in E is defined by:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where functions $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Therefore, the pair $\langle \mu_A(x), \nu_A(x)\rangle$ is an IFP.

Let the universe E be the set N of all natural numbers, starting with 1. Then, we can construct the set

$$N^* = \{\langle n, \frac{\varphi(n)}{\sigma(n)}, \frac{\psi(n) - \varphi(n)}{\sigma(n)}\rangle | x \in E\}.$$

This set is an IFS, because

$$0 \leq \frac{\varphi(n)}{\sigma(n)} + \frac{\psi(n) - \varphi(n)}{\sigma(n)} = \frac{\psi(n)}{\sigma(n)} \leq 1.$$

Therefore, each pair $\langle\frac{\varphi(n)}{\sigma(n)}, \frac{\psi(n)-\varphi(n)}{\sigma(n)}\rangle$ is an IFP. Now, we see that this IFP is a FP if an only if n is square-free.

Also, we can construct the following more complex, than above, IFS:

$$N^{\#} = \{\langle n, \frac{\chi(n)-\varphi(n)}{\sigma(n)}, \frac{\sigma(n)-\psi(n)}{\sigma(n)}\rangle | x \in E\}.$$

Really, $0 \leq \frac{\chi(n)-\varphi(n)}{\sigma(n)} \leq 1,\ 0 \leq \frac{\sigma(n)-\psi(n)}{\sigma(n)} \leq 1$ and from $\varphi(n) \leq \chi(n) < n$ it follows that:

$$0 \leq \frac{\chi(n)-\varphi(n)}{\sigma(n)} + \frac{\sigma(n)-\psi(n)}{\sigma(n)} \leq \frac{\sigma(n)-n+\chi(n)}{\sigma(n)} < 1.$$

Let us call the IFP $\langle 0, 0\rangle$ an uncertain IFP. Now, we see, that each pair $\langle\frac{\chi(n)-\varphi(n)}{\sigma(n)}, \frac{\sigma(n)-\psi(n)}{\sigma(n)}\rangle$ is an IFP that is an uncertain IFP if an only if n is square-free.

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