Conjectured polynomial time primality tests
for numbers of special forms

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Abstract: Conjectured polynomial time primality tests for numbers of special forms similar to
the Riesel primality test for numbers of the form $k \cdot 2^n - 1$ are introduced.

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1 Introduction

In number theory the Riesel primality test [1], is the fastest deterministic primality test for num-
bers of the form $k \cdot 2^n - 1$ with $k$ odd and $k < 2^n$. The test was developed by Hans Riesel and it
is based on Lucas-Lehmer test [2]. In this note I present polynomial time primality tests that are
similar to the Riesel primality test.

2 Main result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left( (x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m \right)$, where $m$ and $x$ are
positive integers.

Conjecture 2.1. Let $N = k \cdot 2^n - 1$ such that $n > 2$, $3 \mid k$, $k < 2^n$ and

\[
\begin{align*}
  k &\equiv 1 \pmod{10} \text{ with } n \equiv 2, 3 \pmod{4} \\
  k &\equiv 3 \pmod{10} \text{ with } n \equiv 0, 3 \pmod{4} \\
  k &\equiv 7 \pmod{10} \text{ with } n \equiv 1, 2 \pmod{4} \\
  k &\equiv 9 \pmod{10} \text{ with } n \equiv 0, 1 \pmod{4}
\end{align*}
\]
Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(3)$, then

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

**Conjecture 2.2.** Let $N = k \cdot 2^n - 1$ such that $n > 2$, $3 \mid k$, $k < 2^n$ and

$$\begin{cases}
    k \equiv 3 \pmod{42} \text{ with } n \equiv 0, 2 \pmod{3} \\
    k \equiv 9 \pmod{42} \text{ with } n \equiv 0 \pmod{3} \\
    k \equiv 15 \pmod{42} \text{ with } n \equiv 1 \pmod{3} \\
    k \equiv 27 \pmod{42} \text{ with } n \equiv 1, 2 \pmod{3} \\
    k \equiv 33 \pmod{42} \text{ with } n \equiv 0, 1 \pmod{3} \\
    k \equiv 39 \pmod{42} \text{ with } n \equiv 2 \pmod{3}
\end{cases}$$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(5)$, then

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

**Conjecture 2.3.** Let $N = k \cdot 2^n + 1$ such that $n > 2$, $k < 2^n$ and

$$\begin{cases}
    k \equiv 5, 19 \pmod{42} \text{ with } n \equiv 0 \pmod{3} \\
    k \equiv 13, 41 \pmod{42} \text{ with } n \equiv 1 \pmod{3} \\
    k \equiv 17, 31 \pmod{42} \text{ with } n \equiv 2 \pmod{3} \\
    k \equiv 23, 37 \pmod{42} \text{ with } n \equiv 0, 1 \pmod{3} \\
    k \equiv 11, 25 \pmod{42} \text{ with } n \equiv 0, 2 \pmod{3} \\
    k \equiv 1, 29 \pmod{42} \text{ with } n \equiv 1, 2 \pmod{3}
\end{cases}$$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(5)$, then

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

**Conjecture 2.4.** Let $N = k \cdot 2^n + 1$ such that $n > 2$, $k < 2^n$ and

$$\begin{cases}
    k \equiv 1 \pmod{6} \text{ and } k \equiv 1, 7 \pmod{10} \text{ with } n \equiv 0 \pmod{4} \\
    k \equiv 5 \pmod{6} \text{ and } k \equiv 1, 3 \pmod{10} \text{ with } n \equiv 1 \pmod{4} \\
    k \equiv 1 \pmod{6} \text{ and } k \equiv 3, 9 \pmod{10} \text{ with } n \equiv 2 \pmod{4} \\
    k \equiv 5 \pmod{6} \text{ and } k \equiv 7, 9 \pmod{10} \text{ with } n \equiv 3 \pmod{4}
\end{cases}$$

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(8)$, then

$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

**Conjecture 2.5.** Let $F = 2^{2^n} + 1$ such that $n \geq 2$. Let $S_i = P_4(S_{i-1})$ with $S_0 = 8$, then

$F$ is prime iff $S_{2^{n-1}-1} \equiv 0 \pmod{F}$

**Conjecture 2.6.** Let $N = k \cdot 2^n - 3$ such that $n > 3$, $k$ is odd $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, then

$N$ is prime iff $S_{n-1} \equiv -P_1(6) \pmod{N}$
Conjecture 2.7. Let $N = k \cdot 2^n + 3$ such that $n > 4$, $k$ is odd $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, then
$N$ is prime iff $S_{n-1} \equiv -P_2(6) \pmod{N}$

Conjecture 2.8. Let $N = k \cdot 2^n - 5$ such that $n > 4$, $k$ is odd $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, then
$N$ is prime iff $S_{n-1} \equiv -P_2(6) \pmod{N}$

Conjecture 2.9. Let $N = k \cdot 2^n + 5$ such that $n > 4$, $k$ is odd $3 \nmid k$, $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(4)$, then
$N$ is prime iff $S_{n-1} \equiv -P_2(4) \pmod{N}$

Conjecture 2.10. Let $N = k \cdot 2^n - 7$ such that $n > 8$, $k$ is odd $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, then
$N$ is prime iff $S_{n-1} \equiv P_4(6) \pmod{N}$

Conjecture 2.11. Let $N = k \cdot 2^n + 7$ such that $n > 6$, $k$ is odd $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, then
$N$ is prime iff $S_{n-1} \equiv P_4(6) \pmod{N}$

Conjecture 2.12. Let $N = k \cdot 2^n - 9$ such that $n > 5$, $k < 2^n$ and
\[
\begin{align*}
  &k \equiv 1 \pmod{6} \text{ with } n \equiv 0 \pmod{2} \\
  &k \equiv 5 \pmod{6} \text{ with } n \equiv 1 \pmod{2}
\end{align*}
\]

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(4)$, then
$N$ is prime iff $S_{n-1} \equiv -P_4(4) \pmod{N}$

Conjecture 2.13. Let $N = k \cdot 2^n + 9$ such that $n > 10$, $k$ is odd $k < 2^n$.

Let $S_i = P_2(S_{i-1})$ with $S_0 = P_k(6)$, then
$N$ is prime iff $S_{n-1} \equiv P_4(6) \pmod{N}$

Conjecture 2.14. Let $N = k \cdot b^a - 1$ such that $n > 2$, $k$ is odd $3 \nmid k$, $b$ is even $3 \nmid b$, $k < b^n$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{6k/2}(P_{b/2}(4))$, then
$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 2.15. Let $N = k \cdot b^n + 1$ such that $n > 2$, $b$ is even $3 \nmid b$, $7 \mid b$, $k < b^n$ and
\[
\begin{align*}
  &k \equiv 5, 19 \pmod{42} \text{ with } n \equiv 0 \pmod{3} \\
  &k \equiv 13, 41 \pmod{42} \text{ with } n \equiv 1 \pmod{3} \\
  &k \equiv 17, 31 \pmod{42} \text{ with } n \equiv 2 \pmod{3} \\
  &k \equiv 23, 37 \pmod{42} \text{ with } n \equiv 0, 1 \pmod{3} \\
  &k \equiv 11, 25 \pmod{42} \text{ with } n \equiv 0, 2 \pmod{3} \\
  &k \equiv 1, 29 \pmod{42} \text{ with } n \equiv 1, 2 \pmod{3}
\end{align*}
\]

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Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b(k/2)}(P_{b/2}(5))$, then
$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

**Conjecture 2.16.** Let $N = k \cdot b^n + 1$ such that $n > 2, b$ is even, $3 \nmid b, 5 \nmid b, k < b^n$ and

\[
\begin{aligned}
& k \equiv 1 \pmod{6} \text{ and } k \equiv 1, 7 \pmod{10} \text{ with } n \equiv 0 \pmod{4} \\
& k \equiv 5 \pmod{6} \text{ and } k \equiv 1, 3 \pmod{10} \text{ with } n \equiv 1 \pmod{4} \\
& k \equiv 1 \pmod{6} \text{ and } k \equiv 3, 9 \pmod{10} \text{ with } n \equiv 2 \pmod{4} \\
& k \equiv 5 \pmod{6} \text{ and } k \equiv 7, 9 \pmod{10} \text{ with } n \equiv 3 \pmod{4}
\end{aligned}
\]

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b(k/2)}(P_{b/2}(8))$, then
$N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$

**Conjecture 2.17.** Let $N = b^n - b - 1$ such that $n > 2, b \equiv 0, 6 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, then
$N$ is prime iff $S_{n-1} \equiv P_{(b+2)/2}(6) \pmod{N}$

**Conjecture 2.18.** Let $N = b^n - b - 1$ such that $n > 2, b \equiv 2, 4 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, then
$N$ is prime iff $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$

**Conjecture 2.19.** Let $N = b^n + b + 1$ such that $n > 2, b \equiv 0, 6 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, then
$N$ is prime iff $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$

**Conjecture 2.20.** Let $N = b^n + b + 1$ such that $n > 2, b \equiv 2, 4 \pmod{8}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(6)$, then
$N$ is prime iff $S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$

**References**
