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Fibonacci primes

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Abstract: Fibonacci composites with prime subscripts, F_p, have factors ($kp \pm 1$) in which k is even. The class of p governs the class of k in the modular ring Z_5, and the digit sum of p, F_p and a function of F_p provide an approximate check on primality.
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1 Introduction

In recent papers [3, 4] methods have been given to characterise Fibonacci primes. Here, we provide a simple method to determine the factors for Fibonacci composites with prime subscripts, F_p. In [3, 4] it was shown that many F_p properties are closely related to the modular ring Z_5 (Table 1) and we find that here too.

Class	$\bar{0}_5$	$\bar{1}_5$	$\bar{2}_5$	$\bar{3}_5$	$\bar{4}_5$
Row	$5r_0$	$5r_1+1$	$5r_2+2$	$5r_3+3$	$5r_4+4$
0	0	1	2	3	4
1	5	6	7	8	9
2	10	11	12	13	14
3	15	16	17	18	19
4	20	21	22	23	24
5	25	26	27	28	29

Table 1. The modular ring Z_5

2 Factors of Fibonacci composites with prime subscripts

The factors were found to have the form ($kp \pm 1$) in which k is even (Table 2) and in a specific class determined by p (Table 3).

p	F_p	*k*	Factors $kp\pm1$	sign
19	4181	2, 6	37, 113	– –
31	1346269	18, 78	557, 2417	– –
37	24157817	2, 4, 60	73, 149, 2221	– + +
41	165580141	68, 1448	2789, 59369	+ +
53	53316291173	18, 1055580	953, 55945741	– +
59	956722026041	6, 4593662	353, 2710260697	– –
67	44945570212853	4, 2493789614	269, 167083904137	+ –
79	14472334024676221	2,1166841411326	157, 92180471494753	– –
89	1779979416004714189	12, 18708857548320	1069, 1665088321800481	+ +

Table 2. Composite Fibonacci numbers

p^*	k^*	Class in Z_5
1	8	$\overline{3}_5$
3	8, 0	$\overline{3}_5\ \overline{0}_5$
7	2, 4, 0	$\overline{2}_5\ \overline{4}_5\ \overline{0}_5$
9	2, 6, 0	$\overline{1}_5\ \overline{2}_5\ \overline{0}_5$

Table 3. Z_5 classes

3 Sum of digits of F_p

The sums of digits of integers, also known as the digital root function, φ(*n*), following [1, 2,6], is given by

$$n = \sum_{i=1}^{k} a_i.10^{k-i} \equiv \overline{a_1 a_2 ... a_k}$$

where a_i is a natural number and $0 \le a_i \le 9$ ($1 \le i \le k$). Then

$$\varphi(n) = \begin{cases} 0, & n = 0, \\ \sum_{i=1}^{k} a_i, & n > 0. \end{cases}$$

although from this point on in this paper it is sufficient to use the decimal count system. Now let us consider the sums of digits of F_p. These give different values for primes and composites except for right-end-digits p^* = 3 or 9 where a sum of 5 is non-distinctive. However, these sums give a rough guide to primality or the possibility of F_p being composite (Table 4).

p^*	Primes sum*	Composites sum*
1	1,2,8	4
3	1,3,5,8	5
7	1,4	5,8
9	5	4,5,8

p^*	Primes sum*	Composites sum*
1	2,9	6
3	3,9	6
7	3,9 (6)	6,9
9	3	3,6,9

Table 4. *sum of digits* Table 5

Another check is to subtract the digital root function $\varphi(F_p)$ from F_p, then divide that number by 3 and sum the digits; that is:

$$\varphi\left(\tfrac{1}{3}\left(F_p - \varphi(F_p)\right)\right)$$

as in Table 5. Generally, composites have a sum of 6, with F_{47} an exception. Even the sum of digits of p show distinctions between primes and composites (Table 6).

p*	**Sum of digits of p**	
	F_p prime	**F_p composites**
1	2,7,8	4,5
3	1,2,4,5,7	8
7	2,7,8	1,4
9	2	1,5,7,8

Table 6. Sums of digits of p

4 Final comments

The first factor of the composites generally has a small value for k so that primality is relatively easy to determine. The digit sums also provide guides to primality in special sequences such as the Fibonacci numbers [5].

The digit sum of an integer follows simply from the integer structure, so we can use this sum to characterise certain identities. For example, perfect numbers, N_p, are represented by $2^{p-1}(2^p - 1)$ where p is a prime (Table 7)

p	**N_p**	**Class of N_p in Z_5**	**$\varphi(N_p)$**
2	6	$\bar{1}_5$	6
3	28	$\bar{3}_5$	1
5	496	$\bar{1}_5$	1
7	8128	$\bar{3}_5$	1
11	2096128	$\bar{3}_5$	1
13	33550336	$\bar{1}_5$	1
17	8589869056	$\bar{1}_5$	1

Table 7. Digits sums of some perfect numbers

This illustrates $N_p \in \{\bar{1}_5, \bar{3}_5\} \Rightarrow N_p^* \in \{6, 8\}$ and $\varphi\{N_p\} = 1, p > 2$. For the Fibonacci primes, another digit sum which can be used comes from a modification of Simson's identity:

$$F_{p-1}F_{p+1} = (F_p - 1)(F_p + 1).$$

The corresponding right-end-digit results are

$$p^* = \begin{cases} 1,9 & \Rightarrow \dfrac{F_p - 1}{p} \in Z, \\ 3,7 & \Rightarrow \dfrac{F_p + 1}{p} \in Z, \end{cases}$$

as in Table 8.

p^*	Sum of digits	
	F_p prime	F_p composites
1	1, 8, 9	3, 6
3	2, 4, 6, 9	3
7	1, 4	6, 9
9	2	3, 4, 5, 6

Table 8. Fibonacci primes digit sums

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