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Professor Anthony Shannon at $F_4 \times F_5 \times F_5$ years

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Time flies very quickly – some days ago – in 1998, in [1], in respect of his 60th birthday, I wrote: *Professor Anthony Shannon, or as all his friends call him - Tony Shannon, has carried out a lot of research in different areas of mathematics: number theory, mathematical modeling, fuzzy sets and others.*

In that paper, I discussed more than 40 papers of his, published in "The Fibonacci Quarterly" in a period of about 30-years. Here, briefly, I will mention the next results of his, published after year 1999.

In the recent years, Tony's research has related to the following topics, ordered chronologically in the References below:

- Topic 1: Properties of Fibonacci Numbers [2, 3, 10, 11, 12, 14, 16, 17, 21, 22, 30, 34, 38, 45]
- Topic 2: Second-order recursive sequences [23, 24, 25, 26, 31, 40, 41, 42]
- Topic 3: Higher-order recursive sequences [29, 33]
- Topic 4: Generalized integers [6, 13, 15, 18, 19, 20, 35]
- Topic 5: Geometric and combinatorial analogies [7, 9, 27, 28, 32, 39, 43, 44, 11]
- Topic 6: Numerical analysis in number theory [4, 5, 8]

The results within the **first** topic include the analysis of Mersenne-Fibonacci and Mersenne–Lucas primes, Fibonacci sequences and the m-square equation, Identities of Hoggatt and Horadam, and inequalities among related pairs of Fibonacci numbers, sets of extensions of the Fibonacci sequence, Fibonacci odd number array, structure of Fibonacci numbers in modular rings, digit sum bases for Fibonacci and related numbers, extremal problems related to the Fibonacci sequence, application of the ψ-function over the members of the Fibonacci sequence.

The **second** topic of research contains results over recurrence relations associated with the Alavi sequence, generalized Fibonacci and Lucas sequences with Pascal-type arrays, Lucas theorem for extended generalized binomial coefficients, an auxiliary equation associated with the

Plastic numbers, relations associated with the Alavi sequence, Pellian sequence relationships among π, e and $\sqrt{2}$, the geometrical and Pellian sequences, and odd-powered triples and Pellian sequences.

The **third** direction of Tony Shannon's research is related to studying of some properties of generalized third order Pell numbers, while, the **fourth** direction contains results on modified Lah numbers, generalized Bernoulli polynomials and Jacksons calculus of sequences, some Fermatian special functions and numbers, a Fermatian Staudt–Clausen theorem, properties of generalized binomial k-extensions, and Fermatian analogues of Goulds generalized Bernoulli polynomials.

The papers included in the **fifth** group are devoted to reflections on the lambda triangle, to a new variant of a Fibonacci plane, to some combinatorial and recurrence relations for shapes in a trellis, to a Fibonacci cylinder, to some recurrence relations for binary sequence matrices, to Pellian sequences and squares and to the Golden ratio.

Finally, the **sixth** group of Tony's research contains results on some aspects of the dominant root of a characteristic polynomial, on expansion of integer powers from Fibonacci odd number triangle, and on combinatorial matrices and linear recursive sequences.

A part of the results on Fibonacci sequence, obtained by the end of the last century by Tony Shannon, are collected in the book [11], written together with Vassia Atanassova, John Turner and me.

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