NNTDM 16 (2010) 3, 25-28

NOTE ON φ, ψ AND σ-FUNCTIONS. PART 2

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Abstract. An interesting property of arithmetic functions φ, ψ and σ is being discussed and illustrated.

Keywords. Arithmetic function φ, Arithmetic function ψ, Arithmetic function σ

AMS Classification. 11A25

In [3, 5] the well known Russian specialist in the area of topology Prof. Anatoly Fomenko introduced (in some cases – with coauthor) the idea that the history of Egypt, Roman (East and West) Empire, Russia and UK are falsified[1]. One of the methods that he used is called by him "pine tree method" also known as "dynastic parallelism". On the two sides of a vertical axis names of the kings of two countries or kings of different dynasties of one country are put together sequentially with the durations of their reigns.

We will not comment on Fomenko's approach to history, but we will use the idea of the pine tree representation for the case of the arithmetic functions φ, ψ and σ, that have the following forms (see, e.g., [4]) for each natural number $n \geq 2$ with a canonical representation

$$n = \prod_{i=1}^{k} p_i^{\alpha_i},$$

where $p_1, p_2, ..., p_k$ are different prime numbers and $\alpha_1, \alpha_2, ...\alpha_k \geq 1$ are natural numbers:

$$\varphi(n) = n.\prod_{i=1}^{k}(1 - \frac{1}{p_i});$$

$$\psi(n) = n.\prod_{i=1}^{k}(1 + \frac{1}{p_i});$$

$$\sigma(n) = \prod_{i=1}^{k} \frac{p_i^{\alpha_i+1} - 1}{p_i - 1}.$$

On Fig. 1 and Fig. 2 we show "pine trees" for pairs (φ, σ) and (φ, ψ). Now, the vertices on the axis are numbered by the natural numbers n. Leftly and rightly from them stay the vertices that correspond to the values of the pair $\varphi(n)$ and $\sigma(n)$ or $\varphi(n)$ and $\psi(n)$. The lengths of the "pine tree branch" are $n - \varphi(n)$ and $\sigma(n) - n$ and $n - \varphi(n)$ and $\psi(n) - n$, respectively.

These two figures show in a new way (for another one see [1, 2]) the close relationship between function φ on the side and functions ψ and σ on the other.

[1] We can mention that similar idea is introduced by Bulgarian mathematician Prof. Jordan Tabov in [6].

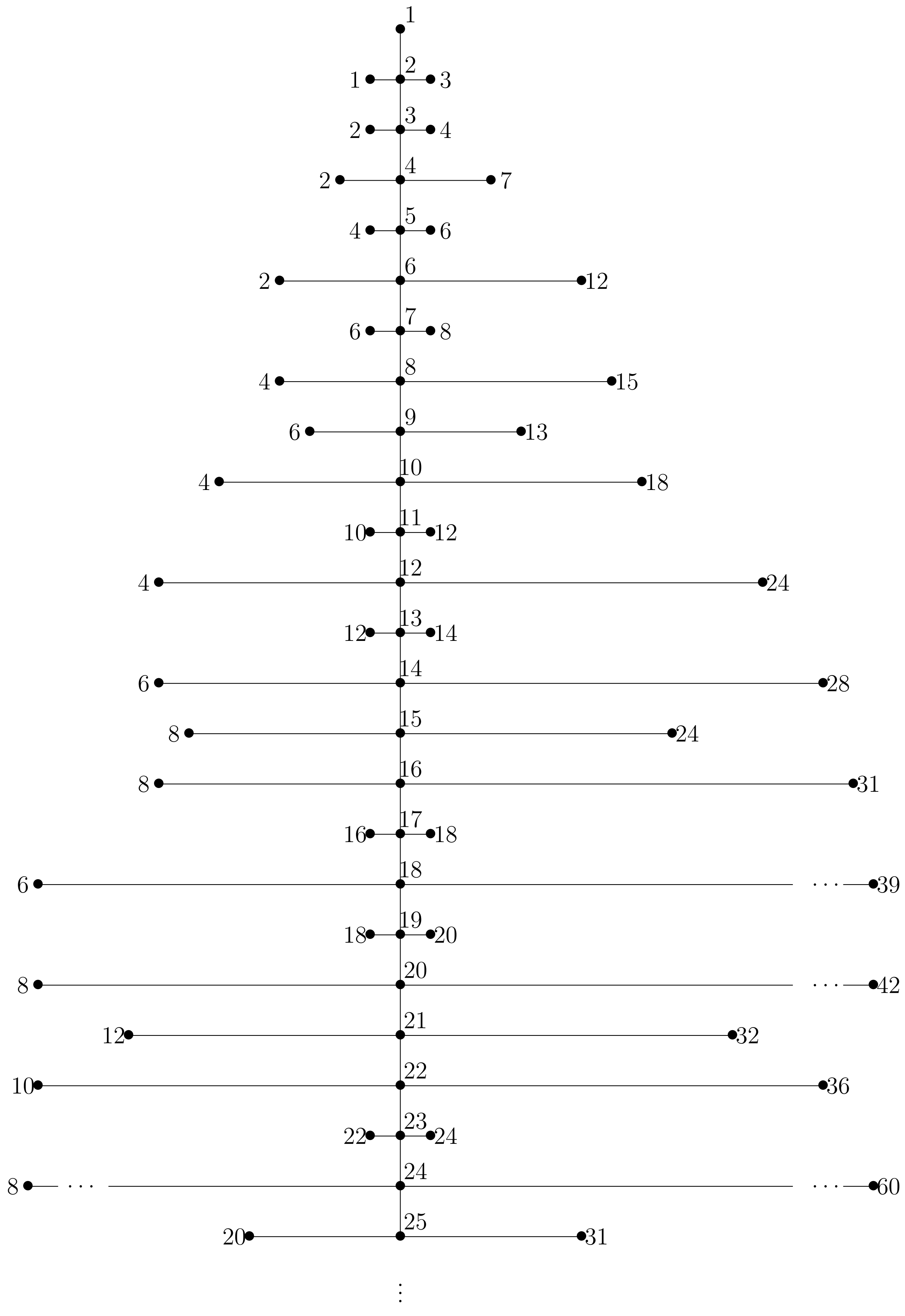

Fig. 1: "alder tree" for φ and σ-functions

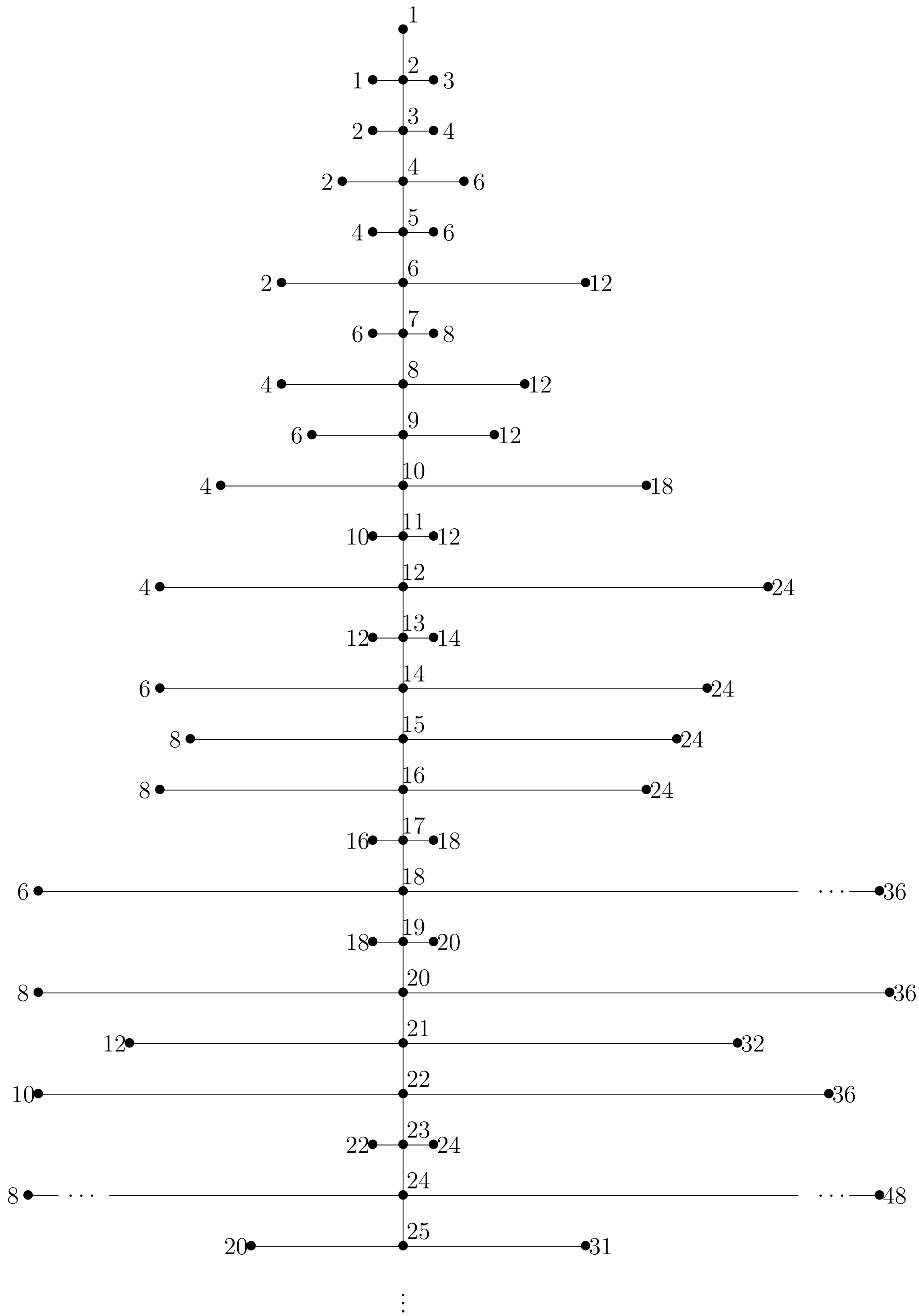

Fig. 2: "alder tree" for φ and ψ-functions

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