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A remark on an arithmetic function. Part 1

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In a series of papers the author studied constructed formulae of n-th prime number p_n, based on some arithmetic functions φ and σ (see, e.g. [3, 4]).

Here a new arithmetic function will be introduced and used to construct a formula for p_n. Probably, this formula will be simpler than the previous ones.

Following notation of [1, 2] we shall define for the natural number $n = \prod\limits_{i=1}^{k} p_i^{\alpha_i}$, where k, α_1, α_2, ... $\alpha_k \geq 1$ are natural numbers and $p_1, p_2, ..., p_k$ are different prime numbers, function:

$$\rho(n) = \begin{cases} 0, & \text{if } n \text{ is a prime number} \\ 1, & \text{otherwise} \end{cases}$$

and

$$\rho(1) = 1.$$

The check of the values of this function is easy. It can stop when the first divisor of n is founded. The number of checks (let us note it by $\nu(n)$) for it for the first 200 natural numbers is given on the Table 1.

Let

$$N(n) = \sum_{i=1}^{n} \nu(n).$$

Hypothesis: For each natural number n

$$N(n) < n\sqrt{n}.$$

Let us define functions sg and $\overline{sg}$ by:

$$sg(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ 1, & \text{if } x > 0 \end{cases},$$

$$\overline{sg}(x) = \begin{cases} 0, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases},$$

where x is a real number.

The relation between functions ρ and ν is

$$\rho(n) = sg(\nu(n) - 1).$$

Following idea from [5, 6], where we introduced four new formulae for well-known function $\pi(n)$ (see, e.g. [3, 4]) and a new formula for the n-th prime number p_n. Now we shall

introduce another – simpler formala for $\pi(n)$ and p_n.
THEOREM 1: The following equality holds for every natural number $n \geq 2$:

$$\pi(n) = n - \sum_{k=1}^{n} \rho(k). \tag{1}$$

Proof: For every natural number k, such that $k \leq n$, if k is prime, then $\rho(k) = 0$, i.e., the sum in (1) corresponds to the number of non-prime numbers that are smaller than n and therefore, the right side of (1) is exactly equal to the number of the prime numbers smaller or equal to n.

Of course, $\pi(0) = 0$ and $\pi(1) = 0$.

Table 1

n	$\nu(n)$	$N(n)$	n	$\nu(n)$	$N(n)$	n	$\nu(n)$	$N(n)$	n	$\nu(n)$	$N(n)$
1	1	1	26	1	67	51	2	172	76	1	311
2	1	2	27	2	69	52	1	173	77	5	316
3	2	4	28	1	70	53	16	189	78	1	317
4	1	5	29	10	80	54	1	190	79	22	339
5	3	8	30	1	81	55	3	193	80	1	340
6	1	9	31	11	92	56	1	194	81	3	343
7	4	13	32	1	93	57	2	196	82	1	344
8	1	14	33	2	95	58	1	197	83	23	367
9	2	16	34	1	96	59	17	214	84	1	368
10	1	17	35	3	99	60	1	215	85	3	371
11	5	22	36	1	100	61	18	233	86	1	372
12	1	23	37	12	112	62	1	234	87	2	374
13	6	29	38	1	113	63	2	236	88	1	375
14	1	30	39	2	115	64	1	237	89	24	399
15	2	32	40	1	116	65	3	240	90	1	400
16	1	33	41	13	129	66	1	241	91	2	402
17	7	40	42	1	130	67	19	260	92	1	403
18	1	41	43	14	144	68	1	261	93	2	405
19	8	49	44	1	145	69	2	263	94	1	406
20	1	50	45	3	148	70	1	264	95	3	409
21	2	52	46	1	149	71	20	284	96	1	410
22	1	53	47	15	164	72	1	285	97	25	435
23	9	62	48	1	165	73	21	306	98	1	436
24	1	63	49	4	169	74	1	307	99	2	438
25	3	66	50	1	170	75	3	310	100	1	439

For the so constructed formula for $\pi(n)$, by analogy with [5, 6] we can prove
THEOREM 2: For every natural number n:

$$p_n = \sum_{i=0}^{C(n)} sg(n - \pi(i)),$$

where (see [7])

$$C(n) = [\frac{n^2 + 3n + 4}{4}].$$

References

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