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A FIBONACCI CYLINDER

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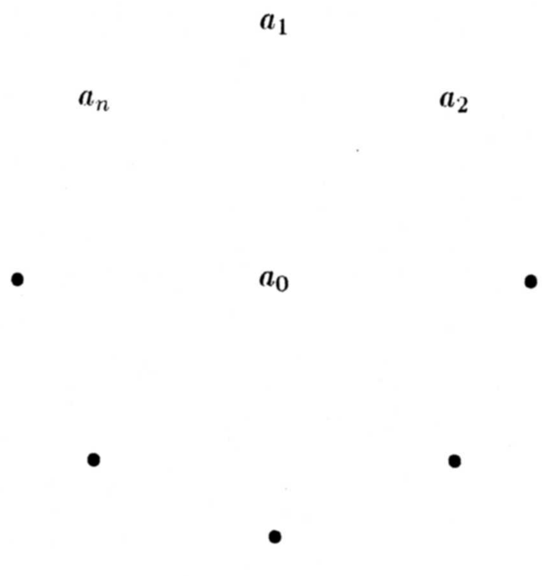
Belogradchik is a small town in the most North-West Bulgaria. Around it there are interesting natural formations - big stones with different strange forms. On 1 May 2007- Day of the Work - and by this reason the mathematicians work, we,the authors visited Belogradchik and the forms of the stones generated idea for a new extension of Fibonaccy sequence. For some of the previous ones see our book [3].

Let us have the real numbers $a_0, a_1, ..., a_n$ and $b_0, b_1, ..., b_n$. Let

$$\alpha = \frac{1}{n}\sum_{i=1}^{n} a_i,$$

$$\beta = \frac{1}{n}\sum_{i=1}^{n} b_i.$$

Let us have a cylinder brocken on levels, so, on the first level stay numbers $a_0, a_1, ..., a_n$, as it is shown:



on the second - numbers $b_0, b_1, ..., b_n$ as it is shown:

$$b_1$$

$$b_n \qquad\qquad b_2$$

$$b_0$$

The numbers in the next levels are obtained as function of the numbers from the two previous level. Below we show the forms of the third, fourth and fifth levels:

$$a_0 + b_1$$

$$a_0 + b_n \qquad\qquad a_0 + b_2$$

$$b_0 + \alpha$$

$$a_0 + b_0 + b_1$$

$$a_0 + b_0 + b_n \qquad\qquad a_0 + b_0 + b_2$$

$$b_0 + \alpha + \beta$$

$$a_0 + 2b_0 + b_1 + \alpha$$

$$a_0 + 2b_0 + b_n + \alpha \qquad\qquad a_0 + 2b_0 + b_2 + \alpha$$

$$a_0 + b_0 + \alpha + 2\beta$$

Let for natural number $k \geq 0$ the elements from the k-th level be:

$$\phi_{k,1}$$

$$\phi_{k,n} \qquad\qquad \phi_{k,2}$$

$$\phi_{k,0}$$

Then for $k \geq 0$ and $1 \leq j \leq n$:

$$\phi_{k+2,0} = \frac{1}{n}\sum_{i=1}^{n} \phi_{k,i} + \phi_{k+1,0},$$

$$\phi_{k+2,j} = \phi_{k+1,j} + \phi_{k,0}.$$

The first ten terms of the sequences $\{\phi_{k,0}\}_{k\geq 0}$ and $\{\phi_{k,j}\}_{k\geq 0}$ for some fixed j $(1 \leq j \leq n)$ are the following

n	$\phi_{k,0}$	$\phi_{k,j}$
0	a_0	b_0
1	b_0	b_j
2	$b_0 + \alpha$	$a_0 + b_j$
3	$b_0 + \alpha + \beta$	$a_0 + b_0 + b_j$
4	$a_0 + b_0 + \alpha + 2\beta$	$a_0 + 2b_0 + \alpha + b_j$
5	$2a_0 + 2b_0 + \alpha + 3\beta$	$a_0 + 3b_0 + 2\alpha + \beta + b_j$
6	$3a_0 + 4b_0 + 2\alpha + 4\beta$	$2a_0 + 4b_0 + 3\alpha + 3\beta + b_j$
7	$4a_0 + 7b_0 + 4\alpha + 6\beta$	$4a_0 + 6b_0 + 4\alpha + 6\beta + b_j$
8	$6a_0 + 11b_0 + 7\alpha + 10\beta$	$7a_0 + 10b_0 + 6\alpha + 10\beta + b_j$
9	$10a_0 + 17b_0 + 11\alpha + 17\beta$	$11a_0 + 17b_0 + 10\alpha + 16\beta + b_j$

Let ψ be the integer function defined for every $k \geq 0$ by:

r	$\psi(6.k + r)$
0	1
1	0
2	-1
3	-1
4	0
5	1

Obviously, for every $m \geq 0$,

$$\psi(m+3) = -\psi(n).$$

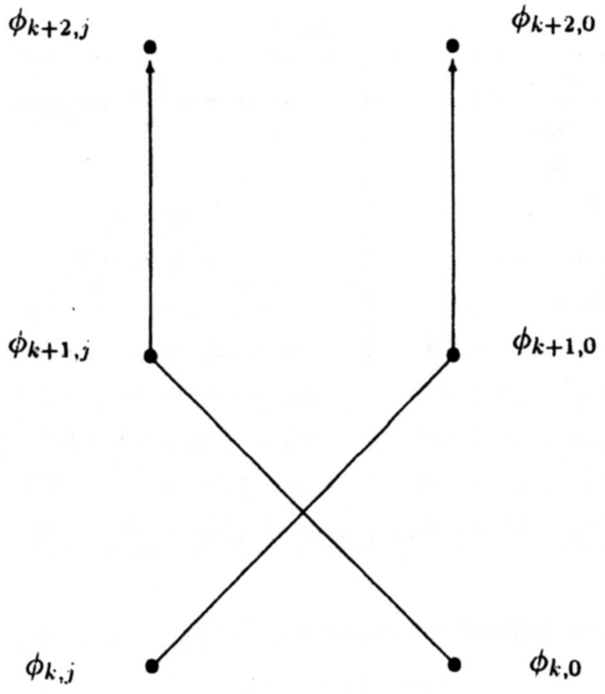
Here we must note that the second schene of 2-Fibonacci sequences, defined in [1.2] is

$$\alpha_0 = a,\ \beta_0 = b,\ \alpha_1 = c,\ \beta_1 = d$$
$$\alpha_{n+2} = \alpha_{n+1} + \beta_n,\ n \geq 0$$
$$\beta_{n+2} = \beta_{n+1} + \alpha_n,\ n \geq 0$$

and its first ten elements are

n	α_n	β_n
0	a	b
1	c	d
2	$b + c$	$a + d$
3	$b + c + d$	$a + c + d$
4	$a + b + c + 2.d$	$a + b + 2.c + d$
5	$2.a + b + 2.c + 3.d$	$a + 3.b + 3.c + 2.d$
6	$3.a + 2.b + 4.c + 4.d$	$2.a + 3.b + 4.c + 4.d$
7	$4.a + 4.b + 7.c + 6.d$	$4.a + 4.b + 6.c + 7.d$
8	$6.a + 7.b + 11.c + 10.d$	$7.a + 6.b + 10.c + 11.d$
9	$10.a + 11.b + 17.c + 17.d$	$11.a + 10.b + 17.c + 17.d$

Now, we see that the vertical section of the cylinder has the form:



Therefore, the sequences $\{\phi_{k,0}\}_{k\geq 0}$ and $\{\phi_{k,j}\}_{k\geq 0}$ for some fixed number j $(1 \leq j \leq n)$ are in an approximately the same relations as the two sequences from 2-Fibonacci sequences. The unique difference is that the members of sequence $\{\phi_{k,j}\}_{k\geq 0}$ contain a fifth component and the coefficient of the fourth component is with 1 smaller that the coefficient of the fourth member of the second sequence in the 2-Fibonacci sequentes.

Let $\{F_k\}_{k\geq 0}$ be the ordinary Fibonacci sequence and let $F_{-1} = 1$.

Hence, it is valid the following

THEOREM: If $k \geq 0, 1 \leq j \leq n$, then the sequences determined the Fibonacci cylinder have the explicit formulae:

$$\phi_{k,0} = \frac{1}{2}.((F_{k-1} + \psi(k)).a_0 + (F_{k-1} + \psi(k+3)).\alpha + (F_k + \psi(k+4)).b_0 + (F_k + \psi(k+1)).\beta)$$

$$= \frac{1}{2}.((a_0 + \alpha).F_{k-1} + (b_0 + \beta).F_k + \psi(k).a_0 + \psi(k+3).\alpha + \psi(k+4).b_0 + \psi(k+1).\beta),$$

$$\phi_{k,j} = \frac{1}{2}.((F_{k-1} + \psi(k+3)).a_0 + (F_{k-1} + \psi(k)).\alpha + (F_k + \psi(k+1)).b_0 + (F_k + \psi(k+4) - 1).\beta) + b_j)$$

$$= \frac{1}{2}.((a_0 + \alpha).F_{k-1} + (b_0 + \beta).F_k + \psi(k+3).a_0 + \psi(k).\alpha + \psi(k+1).b_0 + (\psi(k+4) - 1).\beta + b_j).$$

After Fibonacci speces (see [4]) and Fibonacci pyramid (see [2]), the present construction is the third three-dimensial form of a Fibonacci object.

References

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[3] Atanassov K., V. Atanassova, A. Shannon, J. Turner, New Visual Perspectives on Fibonacci Numbers. World Scientific, New Jersey, 2002.

[4] Atanassov K., Shannon A., Fibonacci planes and spaces, in (F. Howard, Ed.) Applications of Fibonacci Numbers, Vol. 8, Dordrecht, Kluwer, 1999, 43-46.